

**OKLAHOMA Department OF TRANSPORTATION
SPECIAL PROVISIONS
FOR
QUALITY CONTROL AND ACCEPTANCE PROCEDURES FOR
ASPHALT CONCRETE PAVEMENTS (PWL)
IMY-0040-4(391)090, JP NO. 20257(04), CADDO COUNTY**

These Special Provisions revise, amend, and where in conflict, supersede applicable sections of the 1999 Standard Specifications for Highway Construction, English and Metric. Units of measurement are provided in the subsections in both English and Metric equivalents. The units for this project shall be those specified in the project plans. These Special Provisions apply to all types of Asphalt Concrete Pavement.

(add the following:)

411.01 DESCRIPTION.

Contractors Quality Control and Acceptance Procedures will apply to all asphalt as herein specified.

411.04 CONSTRUCTION METHODS.

(m) **Contractor's Quality Control Testing and Inspection.** The Contractor shall provide quality control personnel as necessary to assure the production of quality products as specified. Such personnel shall include one or more Quality Control Technicians who either individually or collectively are fully qualified in the production, placement and testing of plant mix asphalt concrete. Sampling and/or testing of construction materials for either control or acceptance purposes shall be accomplished by persons certified in the appropriate area(s) by the Oklahoma Highway Construction Materials Technician Certification Board.

The Contractor shall be responsible for the formulation of all mix designs. The mix design shall be prepared by an approved asphalt mix design laboratory of the Contractor's choice. All mix designs and changes to the mix designs shall be submitted to the Materials Engineer for review. The Contractor shall perform or have performed all field sampling and testing necessary to ensure that materials and products are within the specified acceptable range. Control charts displaying results of these tests shall be maintained by the Contractor and displayed at the plant site. Copies of the Contractor's quality control tests shall be provided to the Engineer within 24 hours or at time intervals acceptable to the Engineer. Certification by the manufacturers may be used in lieu of field tests when such tests in the field are impracticable. Asphalt cement and additives are examples of materials in this category.

1. *Contractor's Process Control.* The Contractor shall be responsible for the process control of all materials during handling, blending, mixing and placing operations to produce an acceptable asphalt concrete.

At no time will the Engineer issue instructions to the Contractor or producer as to the setting of dials, gauges, scales and meters. However, he/she may advise the Contractor against the continuance of any operations or sequence of operations which will result in non-compliance with Specification requirements.

2. *Contractor's Testing.* For the three characteristics subject to pay adjustments in this Special Provision, the Contractor's sampling and testing shall, as a minimum, comply with the schedule in paragraph (n)(12) "Contractor's Testing and Engineer's Acceptance Procedures". Additional sampling and testing to ensure compliance with Standard Specifications and other Special Provision requirements shall be in accordance with the Contractor's Quality Control Plan.

3. *Contractor's Laboratory.* The Contractor shall provide an approved laboratory at a location no more than 50 road miles from the production site. The laboratory shall be subject to approval of the Engineer.

4. *Contractor's Quality Control Plan.* Prior to initiation of work, the Contractor shall prepare a plan to ensure that acceptable quality can and will be obtained. The plan, which is to be submitted to the Engineer at least one week prior to the prework conference, shall comply with Special Provision 643-6QA and cover all of the items discussed in Section 411 and 708 of the Standard Specifications. However, the Contractor must tailor the plan to meet specific needs of

- the project. Once accepted by the Engineer, the plan becomes a part of the Contract and shall be enforced accordingly. Subsequent changes to the plan may be required by the Engineer in order to adjust to changes in the process or to correct problems in meeting Specification requirements.
5. *Identifying Testing Precision and Bias.* For each test characteristic of each contract item for which control charting is required, the following initialization procedure shall be performed:
- 5.1 Initial testing shall be performed to identify any testing biases between the Contractor's and the Department's testing equipment and procedures. This testing will be referred to as "initialization" testing and will include the plant startup testing as well as the first lot for each contract item for which control charting is required. In all instances, technician certification and equipment identification numbers shall be recorded with each test result to assist in troubleshooting potential testing discrepancies.
- 5.2 The frequency of testing for initialization lots shall be double the normally-specified testing frequency. As such, each initialization lot shall be broken down into ten (10) equal sublots with the sampling within each subplot to be performed at a random location or time interval in accordance with ASTM D 3665 or other acceptable means for ensuring random sampling of the materials. Optionally, the Engineer and Contractor can agree to use the 10 test results from the first two lots for initialization comparisons.
- 5.3 During the initialization testing, for each subplot, the Contractor shall obtain fresh asphalt concrete samples in accordance with AASHTO T 168 under direct observation by the Engineer (for subsequent air voids, asphalt cement content, and gradation testing). The samples shall be split into four (4) equal parts according to AASHTO T 328. The Contractor and the Department shall each perform the following tests (all from the same "split" sample of asphalt concrete): one (1) air voids test and one (1) asphalt cement content test.
- 5.4 During the initialization testing, for each subplot, the Contractor shall obtain a roadway density core from three (3) randomly-selected locations. At each location, one (1) core shall be obtained and tested by each lab. Each of the three (3) locations shall be considered as one test for density. Furthermore, if nuclear methods for determining roadway density are to be used on the project, take the readings over each coring location and correlate each gauge in accordance with OHD L-14. Each core shall be uniquely identified to enable direct comparisons with each lab and, if applicable, with the nuclear density measurements.
- 5.5 During initialization testing, the Contractor and the Engineer may divulge their respective test results before the conclusion of all sampling and associated testing for a given lot and quality characteristic if concerned about the acceptability of the material.
- 5.6 At the conclusion of the sampling and associated testing for a given initialization lot and quality characteristic, a statistical "paired-t test" will be performed by the Engineer (in accordance *Appendix A, Use of Contractor's Test Results for Acceptance Purposes, Part I*) on each characteristic using the pairs of initialization test data. The data should include at least ten (10) pairs each for air voids and asphalt cement content, and thirty (30) pairs for roadway density. The Engineer may, at his discretion, exclude from this and subsequent analyses any initialization test data that are clearly due to explainable special-cause variation provided the cause of said variation has been corrected so as to be unlikely to reoccur.
- 5.7 For those characteristics showing a *statistically* and *practically* significant bias between the Contractor's and the Department's test methods, the following shall govern:
- 5.7.1 The Department's test results shall be relied upon for acceptance and pay adjustment until such time as the source of the bias has been identified and eliminated and the lack of bias subsequently validated in accordance with Appendix A, Part 1, or as provided in Subsection 5.7.2 below.
- 5.7.2 The Contractor may request evaluation of the testing bias via side-by-side three-way testing with an independent-assurance laboratory. In this instance, the following shall govern:
- 5.7.2.1 The Department will select the independent-assurance laboratory, which may be the Department's own independent-assurance laboratory or a third-party laboratory of the Department's choosing.
- 5.7.2.2 The steps outlined in Subsection 5.2. through 5.5. shall be performed utilizing three-way split samples, with the Contractor, Department, and

- independent-assurance laboratory each performing tests on the split-sample specimens.
- 5.7.2.3 At the conclusion of the three-way split-sample testing, three statistical "paired-t tests" will be performed by the Engineer (in accordance with Appendix A, Part 1, using the three pairs of evaluation test data (e.g. Contractor's versus Department's, Contractor's versus independent-assurance laboratory's, and Department's versus independent-assurance laboratory's test results).
- 5.7.2.4 If either of the following conditions are identified as a result of a three way split-sample testing and analysis, Subsection 5.7.2.6 shall govern, otherwise subsection 5.7.2.5 shall govern :
- (i) There is a *statistically- and practically*-significant bias between the Contractor's test methods and the independent-assurance laboratory's test methods, or
 - (ii) the test methods employed by the Contractor during three way split-sample testing are not representative of the test methods to be employed by the Contractor during the normal execution of the project.
- 5.7.2.5 The Contractor shall not be held responsible for any additional costs incurred by the Department in conjunction with the three-way split-sample testing. The Contractor's test results shall be relied upon for acceptance and pay adjustment subject to the provisions of Subsection 5.8.
- 5.7.2.6 The Contractor shall reimburse the Department for all additional costs incurred by the Department as a result of the three-way split-sample testing. The Department's test results shall be relied upon for acceptance and pay adjustment in accordance with Subsection 5.7.1.1.
- 5.8 After the testing precision and bias verification analysis has been performed during the initialization process, ongoing process verification will be performed using the *F*-test and *t*-test. This procedure involves two statistical tests, where the null hypothesis for each test is that the Contractor's test results and the Department's test results are from the same population. The *F*-test provides a method for comparing the variances of the two data sets. The *t*-test provides a method for comparing the means of the two data sets. A significance level of 0.01 shall be used in both tests.
- 5.8.1 Contractor sampling and testing shall follow the guidelines set forth in section (n)(12). The Department shall obtain and test a minimum of one sample per lot, randomly selected and independent of the Contractor's sampling and testing.
- 5.8.2 For the first two lots after initialization only, the test results shall be analyzed using a comparison with the D2S limits. The D2S limits represent the difference between two individual test results that has approximately a five percent chance of being exceeded if the tests are from the same population.
- 5.8.2.1 The absolute difference between the means of the Contractor and Department test results for that lot is compared to the D2S limits. If the absolute test difference is less than or equal to the limit, the two tests are assumed to have come from the same population. The Contractor's results will continue to be used for acceptance.
 - 5.8.2.2 If the test difference exceeds the limit, the two tests are assumed to have come from different populations and the Department will test the remaining half of the Contractor's split samples for acceptance until the reason for the difference is resolved.
- 5.8.3 Beginning with the third lot after initialization, the *F*-test and *t*-test analyses shall be performed as outlined in Appendix A, Part 2.
- 5.8.3.1 For the first 5 lots, all of the test results from each lot are used in both the *F*-test and *t*-test analysis. Afterward, only the results from the current lot plus the previous four lots are used for a total of five lots. In the *F*-test, the mean and variance are computed for the Contractor and Department test results each. The *F* statistic is then calculated by dividing the greater variance by the lesser variance. F_{crit} is then determined using the degrees of freedom from each data

set and compared to the F statistic. If the calculated F statistic is less than or equal to F_{crit} , the two data sets are assumed to have come from the same population. If the calculated F statistic is greater than F_{crit} , the two data sets are assumed to have come from different populations.

5.8.3.2 In the t -test, the Contractor and Department variances are used to calculate the pooled variance, s_p^2 . The t statistic can then be calculated using the means and the pooled variance. t_{crit} is then determined using the degrees of freedom from each data set and compared to the calculated t statistic. If the calculated t statistic is less than or equal to t_{crit} , the two data sets are assumed to have come from the same population. If the calculated t statistic is greater than t_{crit} , the two data sets are assumed to have come from different populations.

5.8.3.3 If both the F -test and t -test conclude that the data sets likely came from the same population, the Contractor's test results continue to be used for acceptance. If one or both the F -test and t -test conclude that the data sets likely came from different populations, the Department's results will be used for acceptance until the source of the bias has been determined and corrected according to subsection 5.7.

6. Control Charts for Quality Control. The Contractor shall maintain and keep current control charts covering, as a minimum, the characteristics of air voids, asphalt cement content, gradation (for each sieve), and roadway density. In all instances, technician and equipment identification numbers shall be recorded with each test to assist in troubleshooting potential testing discrepancies.
 - 6.1 The control charts shall be "individuals" or "individuals and moving range" (I&MR) type control charts or as otherwise approved by the Engineer.
 - 6.2 The charts shall identify the project number; the mix design number; the characteristic being measured; the date, time, lot number, subplot number, technician certification number and equipment identification number for each measurement; the applicable upper and lower control limits (but NOT the specification limits); the Contractor's test results; and any other data needed to facilitate control of the process and identify out-of-control conditions for the process in a timely manner.
 - 6.3 The centerline, standard deviation, and upper and lower control limits for each control chart shall initially be calculated based on the initialization test results for each characteristic and contract item.
 - 6.4 An "out of control" condition is defined as the condition resulting from any one of the following eight (8) "alarm" conditions occurring on a single control chart:
 - 6.4.1 Any one point is more than 3 standard deviations from the centerline.
 - 6.4.2 Nine points in a row are on the same side of the centerline.
 - 6.4.3 Six points in a row are all increasing or all decreasing.
 - 6.4.4 Fourteen points in a row are alternating up and down.
 - 6.4.5 Two out of three points are more than 2 standard deviations from the centerline (and on the same side of the centerline).
 - 6.4.6 Four out of five points are more than 1 standard deviation from the centerline (and on the same side of the centerline).
 - 6.4.7 Fifteen points in a row are all within 1 standard deviation of the centerline.
 - 6.4.8 Eight points in a row are all more than 1 standard deviation from the centerline (on either side of the centerline).
 - 6.5 Whenever an out-of-control condition corresponding to alarm criteria 6.4.1, 2, 3, 5, 6 or 8 is observed for any of the control charts, the Contractor shall provide written notification to the Engineer concerning said out-of-control condition within 18 hours of the time the alarm-generating test was performed. In addition, the Contractor shall provide written notification to the Engineer (within 36 hours of the time the alarm-generating test was performed) concerning the investigative and/or corrective actions taken or to be taken. **Failure to provide written notification to the Engineer within the time periods specified shall result in an automatic one-half-percent reduction in the composite pay factor for the affected lot for each failure to comply with the specified notification procedures.** After the probable-cause investigation for the out-of-control condition has been completed, written

notification shall be provided to the Engineer stating the probable cause and corrective actions taken or to be taken to reduce the likelihood of reoccurrence (or, stating that the probable cause could not be determined, if such is the case). The Engineer may, at his discretion, exclude from pay-adjustment calculations any out-of-control test data for which the special-cause variation has been adequately identified and explained, provided the cause of said variation has been corrected so as to be unlikely to reoccur. (NOTE: Not all out-of-control conditions will require corrective action. For example, a change in roadway densities due to a change in rolling pattern may not require corrective action if the densities being obtained remain safely within the specified limits. However, all out-of-control conditions shall be investigated for probable cause, regardless of the proximity of the measured values to the specification limits.)

- 6.6 In the event a significant change to the process occurs or is observed (e.g. change in raw material sources, change in mix proportions, steady drift in roadway densities, etc.), the control limits (including centerline and standard deviation) for the affected characteristics shall be recalculated using the available test data that best represent the new process. Any such changes to control limits must be approved in writing by the Engineer.
- (n) **Acceptance.** While the Contractor shall be fully and exclusively responsible for producing an acceptable product, acceptance responsibility rests with the Engineer. The entire lot of asphalt as defined in paragraph (n) (12) "Lot and Sublot Selection" will be accepted or rejected and paid for on the basis of acceptance test results.

1. *General.* The following characteristics will be considered when determining the acceptability and pay factors for Plant Mix Asphalt Concrete Pavement. However, all of the requirements of the Standard Specifications on materials and workmanship except those superseded by Special Provisions in this Contract, shall remain in effect.

- (a) Asphalt Cement Content
- (b) Air Voids
- (c) Roadway Density

Several methods are available to test for the above characteristics. While only one method will be used, several tests may be made to measure each characteristic. Applicable pay factors will be considered individually in determining payment. Pay factors for asphalt cement content, air voids, and roadway density will apply to all asphalt concrete placed.

2. *Criteria for Lot Acceptance and Payment.* Except for surface smoothness, conformance with the specifications will be judged on the basis of the following criteria:

- 2.1 The estimated Percent-within-Limits (PWL) with respect to asphalt cement content, air voids, and roadway density. The PWL with respect to a particular quality characteristic is the amount of materials and construction which falls within the specified limits listed in the following tables (where "JMF" refers to the corresponding values from the Job Mix Formula):

Quality Characteristic	Lower Specification Limit (LSL)	Upper Specification Limit (USL)
Asphalt Cement Content	JMF - 0.4 %	JMF + 0.4%
Air Voids (lab molded)	JMF ¹ - 1.35 %	JMF ¹ + 1.35 %
Roadway Density	91.5%	97%

Quality Characteristic	Lower Target Limit (LTL)	Upper Target Limit (UTL)
Asphalt Cement Content	JMF - 0.16 %	JMF + 0.16 %
Air Voids (lab molded)	JMF ¹ - 0.75 %	JMF ¹ + 0.75 %
Roadway Density	93%	96%

- 1) Note: JMF=(100 - mid-point of density range shown on the mix design) e.g. JMF for NMS mixtures, 100-96=4.
3. *Acceptable and Rejectable Quality Levels.* A lot shall be considered of acceptable quality with

respect to a particular characteristic if the PWL is no less than 90 percent. A lot shall be considered of rejectable quality with respect to a particular characteristic if the PWL is less than 50 percent. Lots exceeding the Acceptable Quality Level shall be subject to positive pay adjustments as defined in Subsection (n)(5). Lots exceeding the Rejectable Quality Level but falling below the Acceptable Quality Level shall be subject to negative pay adjustments as defined in Subsection (n)(5). If a lot fails to exceed the Rejectable Quality Level in one or more characteristics, the engineer may require its removal and replacement at the Contractor's expense. If this option is not exercised, the Contractor may elect to remove and replace the lot (at his expense) or leave it in place, subject to a pay factor (for that quality characteristic) of zero.

3.1.1.1 The Contractor shall perform the necessary quality-control sampling and testing to ensure that acceptable quality level requirements are consistently met.

4. *Determination of Percent-within-Limits (PWL).* The PWL with respect to each of the characteristics of asphalt content, air voids, and roadway density, will be determined as follows:
- 4.1 Compute the sample mean (\bar{X}) and the sample standard deviation (S') of the n test results (X_i):

$$\bar{X} = \frac{\sum X_i}{n} \quad S' = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}}$$

- 4.2 If \bar{X} falls outside the target limits (LTL and UTL) **and** inside the specification limits (LSL and USL), compute the target-adjusted standard deviation (S'') as follows:

$$S'' = \sqrt{S'^2 + (X_{\text{target}} - \bar{X})^2}$$

4.2.1.1 where X_{target} = the nearest target limit (LTL OR UTL)

- 4.3 If \bar{X} falls inside the target limits (LTL and UTL) or outside the specification limits (LSL and USL), compute the target-adjusted standard deviation (S'') as follows:

$$S'' = S'$$

- 4.4 Using the target-adjusted standard deviation (S''), compute the upper quality index (Q_U) and the lower quality index (Q_L) corresponding to the upper and lower specification limits listed in Subsection (n) (2):

$$Q_U = \frac{USL - \bar{X}}{S''} \quad Q_L = \frac{\bar{X} - LSL}{S''}$$

- 4.5 Using the Percent Defective tables (for sample size $n=5$ or the appropriate table for other values of n), determine the percentage of materials and construction falling outside the specification limits PD_U and PD_L associated with Q_U and Q_L , respectively. Add these two values to obtain the lot percent defective (PD):

$$PD = PD_U + PD_L$$

- 4.6 Determine the percentage of materials and construction falling within the specification limits PWL as follows:

$$PWL = 100 - PD$$

5. *Pay Factors for Lot Quality Characteristics.* Except for pavement smoothness, the pay factor (PF) for each quality characteristic will be determined as follows:

- 5.1 If PWL is greater than or equal to 50 percent, compute the pay factor using the equation:

$$PF = 0.024(PWL) - 0.0001(PWL)^2 - 0.35$$

- 5.2 If PWL is less than 50 percent, the Engineer may require removal and replacement of the defective lot at the Contractor's expense. If this option is not exercised, the Contractor may elect to remove and replace the lot (at his expense) or leave it in place subject to a pay factor (for that quality characteristic) of $PF = 0$.

6. *Pay Adjustment for Lots.* Once a lot has been defined, its identity will be maintained throughout the mixing and placement process. When the lot is completed, the individual pay factors for asphalt content, air voids, and roadway density will be used to calculate a composite pay factor (CPF) and a pay adjustment (PA) for the subject lot as follows:

$$CPF = \frac{5PF_D + 3PF_V + 2PF_A}{10}$$

where:

- PF_A = Pay factor for asphalt content,
 PF_V = Pay factor for air voids, and
 PF_D = Pay factor for roadway density.

The pay adjustment for the completed lot will be determined in accordance with the following formula:

$$PA_{Lot} = (CPF - 1)(CUP)(Q_{Lot})$$

where:

- PA_{Lot} = Pay adjustment for the lot,
 CPF = Composite pay factor,
 CUP = Contract unit price (\$/Ton (\$/Metric Ton)), and
 Q_{Lot} = Quantity of asphalt concrete in the lot (Tons (Metric Tons))

7. *Smoothness Acceptance and Pay Adjustment.* For smoothness determination and pay adjustment purposes, the pavement surface will be tested on an extent-to-extent basis in accordance with Special Provisions 430-2QA. Acceptance and pay adjustment determinations made under Special Provisions 430-2QA will be completely independent of those made under this Special Provision.
8. *Pay Adjustments Not Covered in Special Provisions 411-9QA or 430-2QA.* Adjustments in pay, for deviations from specified standards for characteristics other than those described in these Special Provisions (if any) will be made in accordance with General Provision 105.03.
9. *Total Pay Adjustment for Entire Project.* The total adjustment in pay for the entire project is the sum

of: (1) the pay adjustments for individual lots per Subsection (n)(6); plus (2) the pay adjustments for smoothness per Special Provision 430-2QA; plus (3) other pay adjustments, if appropriate, per Subsection (n)(8).

10. *Extreme Values (Outliers)*. Test results apparently inconsistent with the results of the majority of tests will also be closely examined by the Engineer in order to determine their validity. The examination will cover the procedures used in sampling and testing and, if necessary, a mathematical analysis performed in accordance with ASTM E 178 (upper 2.5% significance level). Test results thus determined by the Engineer to be an outlier may be discarded. The Engineer will determine whether the outlier is representative of the material being evaluated. The remaining test results will then be supplemented, if necessary.
11. *Lot and Sublot Selection*. The asphalt concrete will be randomly sampled and tested for all control test characteristics on a lot to lot basis in accordance with the following requirements. However, any load of mixture which is visually unacceptable for reasons of being excessively segregated or aggregate improperly coated will be rejected for use in the work. Excessively high or low temperature will be cause for rejection. Furthermore, sections of completed pavement which form visual observation or known deficiencies that appear to be seriously inadequate will be tested. The results of such tests will not be used for pay adjustment purposes but will be used to determine whether the section is totally unacceptable and must be removed. In the event that it is determined to be unacceptable, its removal and replacement shall be at no additional cost to the Department. A standard size lot at the asphalt plant shall consist of five (5) equal sublots of 1,000 tons (metric tons) each. Subject to prior notification, the Contractor may reduce the lot size to 3,750 tons (metric tons) or increase it to 6,250 tons (metric tons). Any partial lot (one with less than 3,750, 5,000, 6,250 tons (metric tons)) shall be treated as a separate lot when four (4) or more sublots exist. When a lot contains three (3) or less sublots, it shall be combined with the previous lot. On multiple project contracts, the lots of the asphalt will carry over from project to project within that contract. All acceptance testing shall be performed at a random location or time interval within each sublot in accordance with ASTM D 3665 or other acceptable means for ensuring random sampling of the materials.
12. *Contractor's Testing and Engineer's Acceptance Procedures*. Once a lot has been defined, its identity will be maintained throughout the mixing and placement process. Pay factors, determined from random sampling and testing the lot at appropriate locations will be used in computing its pay adjustment.
 - The Contractor is required as a minimum to comply with the following schedule for sampling and testing. Depending upon the available time and his confidence in the Contractor's Process Control, the Engineer may elect to perform more or less sampling and testing.
 - Asphalt Cement Content and Gradation - one (1) specimen randomly selected per sublot.
 - Air voids (except NMS mixes) - one (1) test per sublot randomly selected. Three (3) specimens shall be averaged and considered as one (1) test.
 - Air voids (NMS mixes) - one (1) test per sublot randomly selected. Two (2) specimens shall be averaged and considered as one (1) test.
 - Roadway Density - three (3) specimens per sublot randomly selected and considered as three (3) tests.
- (o) **Plant Startup Requirements**. Prior to beginning production of asphalt for the mainline the Contractor shall provide a quality control system. The system shall include the fully equipped laboratory and the full complement of quality control personnel that are to perform the quality control functions of the remainder of the project.
 - Plant startup production shall be limited to that necessary to calibrate the plant and the testing equipment and procedures using the mix design approved for mainline construction. The asphalt concrete thus produced shall be sampled and tested by both the Contractor and the Engineer for VMA, gradation and all of the characteristics except roadway density. The Contractor's test results shall then be reconciled with those from the Engineer.

No asphalt concrete from the startup operation shall be placed on the mainline or the control strip. Instead, adjustments shall continue to be made until all of the requirements are met. Asphalt concrete from the plant startup operation may be utilized and paid for in the construction of temporary facilities or if no temporary facilities are available they shall become the property of the Contractor and will not be paid for. Costs associated with startup operations will not be measured separately for payment but will be included in the payment for Contractor's Quality Control.

- (p) **Control Strip Requirements.** After fulfilling the plant startup requirements, one or more control strips shall be constructed on the detour (if available), shoulder (if detour not available) or mainline (if neither detour nor shoulder is available) for the purpose of verifying the required production mix characteristics and establishing rolling patterns to obtain target requirements. The initial placement of asphalt shall be limited to approximately 500 tons (metric tons) plus or minus 100 tons (metric tons). This material shall then be sampled and tested by the Contractor and the Engineer for VMA, gradation and all of the characteristics. No additional asphalt shall be placed until all the results are evaluated and necessary adjustments in production and placement procedures are made. No pay adjustments will be made for deviations from target on the approximately 500 tons (metric tons) placement. However no asphalt in this or any subsequent control strips, which is determined by Special Provision 411-11 to be unacceptable shall be allowed to remain in the mainline or the shoulder. Such unacceptable asphalt shall be removed and replaced by the Contractor at no additional expense to the Department.

After necessary adjustments are made, the above process shall be repeated for the next approximately 500 tons (metric tons) of asphalt placed. Pay adjustments for deviations from target on this second placement will be made at the rate of one half of those specified. If required, additional control strips shall be made until an acceptable product (i.e., within the 1.00 pay factor range) is produced. Pay adjustments for deviations from target on all asphalt after the second placement will be made at the rate specified in Subsection 411.04(n)(9). Control strips will be paid for at the contract unit price (as adjusted) for the appropriate type of asphalt concrete.

Percent Defective Table - Sample Size N=30*

Q	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	50.00	49.60	49.21	48.81	48.42	48.02	47.63	47.24	46.84	46.45
0.1	46.05	45.66	45.27	44.88	44.48	44.09	43.70	43.31	42.92	42.53
0.2	42.15	41.76	41.37	40.99	40.60	40.22	39.83	39.45	39.07	38.69
0.3	38.31	37.93	37.55	37.18	36.80	36.43	36.05	35.68	35.31	34.94
0.4	34.58	34.21	33.85	33.48	33.12	32.76	32.40	32.04	31.69	31.33
0.5	30.98	30.63	30.28	29.93	29.59	29.24	28.90	28.56	28.22	27.89
0.6	27.55	27.22	26.89	26.56	26.23	25.90	25.58	25.26	24.94	24.62
0.7	24.31	23.99	23.68	23.37	23.07	22.76	22.46	22.16	21.86	21.57
0.8	21.27	20.98	20.69	20.40	20.12	19.84	19.56	19.28	19.00	18.73
0.9	18.46	18.19	17.92	17.66	17.40	17.14	16.88	16.63	16.37	16.12
1.0	15.88	15.63	15.39	15.15	14.91	14.67	14.44	14.21	13.98	13.75
1.1	13.53	13.31	13.09	12.87	12.66	12.45	12.24	12.03	11.82	11.62
1.2	11.42	11.22	11.03	10.84	10.64	10.46	10.27	10.09	9.90	9.72
1.3	9.55	9.37	9.20	9.03	8.86	8.69	8.53	8.37	8.21	8.05
1.4	7.90	7.74	7.59	7.44	7.30	7.15	7.01	6.87	6.73	6.60
1.5	6.46	6.33	6.20	6.07	5.95	5.82	5.70	5.58	5.46	5.34
1.6	5.23	5.12	5.01	4.90	4.79	4.68	4.58	4.48	4.38	4.28
1.7	4.18	4.09	3.99	3.90	3.81	3.72	3.63	3.55	3.47	3.38
1.8	3.30	3.22	3.15	3.07	2.99	2.92	2.85	2.78	2.71	2.64
1.9	2.57	2.51	2.45	2.38	2.32	2.26	2.20	2.14	2.09	2.03
2.0	1.98	1.93	1.87	1.82	1.77	1.73	1.68	1.63	1.59	1.54
2.1	1.50	1.46	1.42	1.38	1.34	1.30	1.26	1.22	1.19	1.15
2.2	1.12	1.09	1.05	1.02	0.99	0.96	0.93	0.90	0.88	0.85
2.3	0.82	0.80	0.77	0.75	0.72	0.70	0.68	0.66	0.63	0.61
2.4	0.59	0.57	0.56	0.54	0.52	0.50	0.48	0.47	0.45	0.44
2.5	0.42	0.41	0.39	0.38	0.37	0.35	0.34	0.33	0.32	0.30
2.6	0.29	0.28	0.27	0.26	0.25	0.24	0.23	0.22	0.22	0.21
2.7	0.20	0.19	0.18	0.18	0.17	0.16	0.16	0.15	0.14	0.14
2.8	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10	0.09	0.09
2.9	0.09	0.08	0.08	0.08	0.07	0.07	0.07	0.06	0.06	0.06
3.0	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04
3.1	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02
3.2	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01
3.3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
3.4	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
3.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

* Values in the body of this table are estimates of the lot percent defective corresponding to specific values of Q, the quality Index. For values of Q greater than or equal to zero, the estimate of percent defective is read directly from the table. For negative values of Q, the values read from the table must be subtracted from 100.

Use of Contractor's Test Results for Acceptance Purposes

Part 1: Guidelines for Initial Validation of Contractor's Test Methods

In order to utilize the Contractor's material test results for acceptance and payment, the Department must ensure that the Contractor's results compare favorably with the Department's test results for the same material and same quality characteristic. The following procedure can be used to determine the initial validity of the Contractor's test methods:

1. A "paired test" (as referred to herein) will be any time two (2) separate tests are conducted by the Department and the Contractor (one test each) where the material tested has been collected as a true split sample or was sampled at the same time, from the same lot, subplot and batch, and from the same proximity within the batch. For asphalt air voids, asphalt cement content, and gradation testing, "paired" tests are those tests performed on split samples or samples taken from the same batch and from the same proximity within the batch. For roadway density and thickness measurements from cores, "paired" tests are those measurements performed on the exact same cores. For concrete unit weight, slump, and air content, "paired" tests are those conducted on samples taken from the same batch and from the same proximity within the batch. For concrete, the compressive strength "paired" test will generally be the average strength of two or three specimens each (by the Department and the Contractor). The specimens are cast from concrete sampled from the same batch and from the same proximity within the batch.
2. For each quality characteristic for which the Contractor's test methods are to be validated, the Department will conduct a minimum of ten (10) initial paired tests with the Contractor.
3. The Engineer may, at his discretion, exclude from this and subsequent analyses any paired test data that clearly exhibit explainable special-cause variation provided the cause of said variation has been identified and corrected so as to be unlikely to reoccur.
4. For each quality characteristic, calculate the paired-t test statistic (t_p), the average of the paired differences (\bar{X}_p) and the standard deviation of the paired differences (S_p) as follows:

$$t_p = \left| \sqrt{n_p} \frac{\bar{X}_p}{S_p} \right|$$

Equation (1)

$$\bar{X}_p = \frac{\sum_{i=1}^{n_p} (X_c - X_d)}{n_p}$$

Equation (2)

$$S_p = \sqrt{\frac{\sum_{i=1}^{n_p} (X_c - X_d - \bar{X}_p)^2}{n_p - 1}}$$

Equation (3)

where:

- t_p = Paired-t test statistic. This value will be compared to a critical t-value (t_{crit}) to be obtained from Table 1.
- n_p = The number of paired tests conducted. Each “paired test” consists of two individual tests (one by the Contractor and one by the Department) performed on a single split sample.
- \bar{X}_p = The average of the differences between the paired tests.
- S_p = The standard deviation of the differences between the paired tests. The correct order for computing S_p is to calculate all the differences between the paired tests, then take the standard deviation of those differences.
- X_c = The Contractor’s individual test result for a given split sample.
- X_d = The Department’s individual test result for a given split sample.
- n_p-1 = The number of degrees of freedom for use with Table 1.

5. Obtain the critical t-value t_{crit} , from Table 1, using n_p-1 as the number of degrees of freedom.
6. Compare the calculated paired-t statistic (t_p) to the corresponding critical value from Table 1 (t_{crit}).
7. If t_p is less than t_{crit} , the Contractor’s test methods for that quality characteristic are without significant bias. The Contractor’s test methods for that quality characteristic can be considered valid at the present time. Proceed to Step 12.
8. If t_p is greater than or equal to t_{crit} , there exists a *statistically*-significant bias ($= \bar{X}_p$) between the Contractor’s and the Department’s test methods.
9. Compare the Contractor’s testing bias (\bar{X}_p) to the relevant Allowable Testing Bias (ATB) from the following table:

PCC Quality Characteristic	Allowable Testing Bias (ATB)	Units
Gradation:		
Sieve # 200 (Coarse)	± 0.40	% -passing
Sieve # 200 (Fine)	± 0.30	% -passing
Unit Weight	± 0.80	pcf
Slump	± 0.30	inch
Air Content	± 0.30	% (by volume)
Compressive Strength	± 100	psi
AC Quality Characteristic		
Gradation: ¹		
Sieves #4 and larger	± 1.50	% -passing
Sieves #10 through #80	± 1.00	% -passing
Sieve # 200	± 0.50	% -passing
Asphalt Cement Content	± 0.15	% (by weight)
Air Voids (LMS)	± 0.50	% (by volume)
Roadway Density	± 0.50	% max. theor.

10. If the magnitude of \bar{X}_p is less than the magnitude of the corresponding ATB, the Contractor’s bias for that quality characteristic, though *statistically*-significant, is not practically significant. The Contractor’s test methods for that quality characteristic can be considered valid at the present time. Proceed to Step 12.

¹ ATB’s for gradations are for information and control charts only.

11. If the magnitude of \bar{X}_p is greater than or equal to the magnitude of the corresponding ATB, the Contractor's bias for that quality characteristic is practically significant and unacceptable. The Contractor's test results for that quality characteristic will not be used for acceptance unless and until the Contractor's test methods are validated as follows:
 - (1) The respective test equipment used during the paired testing (both the Contractor's and the Department's) shall be inspected and tested for calibration by a qualified independent calibration specialist.
 - (2) A calibration test certificate shall be prepared by the calibration specialist identifying the pre-calibration errors at various measurement levels such that the paired test data are fully bracketed by the pre-calibration error estimates.
 - (3) The paired test results will then be adjusted by the Engineer so as to effectively nullify the errors identified on each calibration test certificate.
 - (4) Return to Step 4, using the adjusted paired test data in lieu of the original paired test data.

12. Once the Contractor's test methods for a given quality characteristic have been validated, the Engineer will provide ongoing validation of the Contractor's test method in accordance with the *Guidelines for Ongoing Validation of Contractor's Test Methods*.

Table 1 – Critical Values for the Paired-t Test Statistic (based on $\alpha = 0.01$)

Degrees of Freedom ($n_p - 1$)	Critical t-value (t_{crit})	Degrees of Freedom ($n_p - 1$)	Critical t-value (t_{crit})	Degrees of Freedom ($n_p - 1$)	Critical t-value (t_{crit})
2	9.925	25	2.787	48	2.682
3	5.841	26	2.779	49	2.680
4	4.604	27	2.771	50	2.678
5	4.032	28	2.763	60	2.660
6	3.707	29	2.756	70	2.648
7	3.499	30	2.750	80	2.639
8	3.355	31	2.744	90	2.632
9	3.250	32	2.738	100	2.626
10	3.169	33	2.733	110	2.621
11	3.106	34	2.728	120	2.617
12	3.055	35	2.724	130	2.614
13	3.012	36	2.719	140	2.611
14	2.977	37	2.715	150	2.609
15	2.947	38	2.712	160	2.607
16	2.921	39	2.708	170	2.605
17	2.898	40	2.704	180	2.603
18	2.878	41	2.701	190	2.602
19	2.861	42	2.698	200	2.601
20	2.845	43	2.695	300	2.592
21	2.831	44	2.692	400	2.588
22	2.819	45	2.690	500	2.586
23	2.807	46	2.687	1000	2.581
24	2.797	47	2.685	10000	2.576

Use of Contractor's Test Results for Acceptance Purposes

Part 2: Guidelines for Ongoing Validation of Contractor's Test Methods

After the Department has initially validated the Contractor's test method for a particular quality characteristic as outlined in the Guidelines for Initial Validation of Contractor's Test Methods, ongoing process verification will be performed using the F -test and t -test. This procedure involves two statistical tests, where the null hypothesis for each test is that the Contractor's test results and the agency's test results are from the same population. The F -test provides a method for comparing the variances of the two data sets. The t -test provides a method for comparing the means of the two data sets. A significance level of 0.01 shall be used in both tests. The steps for performing the F -test and t -test are as follows:

1. Contractor sampling and testing shall follow the guidelines set forth in section (n)(12). The Contractor shall obtain and split samples. Half the split sample shall be tested by the Contractor and the other half stored for potential Department testing. The Department shall obtain and split a minimum of one sample per lot, randomly selected and independent of the Contractor's sampling and testing. One fourth of the split sample shall be tested by the Department, one fourth stored for potential Contractor testing, and one half stored for potential independent verification.
2. For each of the first two lots after initialization, compare the sample results from the Contractor and Department. If the following is true, the two tests are assumed to have come from the same population and the Contractor's results will continue to be used for acceptance.

$$|\bar{X}_c - \bar{X}_d| \leq D2S$$

where:

\bar{X}_c = Average of the Contractor's test results,

\bar{X}_d = Department's average test result, and

D2S = Acceptable range for 2 tests.

If the following or opposite to that above is true instead, the two tests are assumed to have come from different populations and the Department's results will then be used for acceptance until the source of the bias has been determined and corrected according to subsection 5.7.

$$|\bar{X}_c - \bar{X}_d| > D2S$$

Quality Characteristic	D2S Limits
Asphalt Cement Content	0.30%
Air Voids (lab molded)	1.40%
Roadway Density	1.40%

- Beginning with the third lot after initialization, the F -test and t -test analysis will be used for process verification. By the third lot after initialization, there should be fifteen Contractor results and three Department results. For the first five lots after initialization (including the two used for the D2S comparison), all the test results from each lot are used in both the F -test and t -test analysis. Afterward, only the results from the current lot plus the previous four lots are used for a total of five lots.
- Beginning with the sixth lot after initialization, determine the five lot cumulative mean and variance of the Contractor's test results for each quality characteristic. Determine the five lot cumulative mean and variance of the Department's independently sampled test results for each quality characteristic.
- Calculate the mean and variance of each test set:

$$\bar{X}_c = \frac{\sum X_c}{n_c} \qquad s_c^2 = \frac{\sum (\bar{X}_c - X_c)^2}{n_c - 1}$$

$$\bar{X}_d = \frac{\sum X_d}{n_d} \qquad s_d^2 = \frac{\sum (\bar{X}_d - X_d)^2}{n_d - 1}$$

where:

- \bar{X}_c = Mean of the Contractor's results,
- X_c = Individual Contractor test result,
- n_c = Number of Contractor test results,
- $n_c - 1$ = Degrees of freedom of the Contractor's tests,
- s_c^2 = Variance of the Contractor's results,
- \bar{X}_d = Mean of the Department's result,
- X_d = Individual Department test result,
- n_d = Number of Department test results,
- $n_d - 1$ = Degrees of freedom of the Department's tests, and
- s_d^2 = Variance of the Department's results.

6. Calculate the F-statistic using the larger of the variances in the numerator:

$$F = \frac{s_c^2}{s_d^2} \quad \text{or} \quad F = \frac{s_d^2}{s_c^2}$$

7. Determine the F_{crit} value from the F-distribution table using a 0.01 significance level () and making sure to use the correct degrees of freedom for both the numerator and the denominator.
8. If $F \leq F_{crit}$, the data is assumed to have come from the same population and the Contractor's results will continue to be used for acceptance if the t -test results also indicate that the results have also come from the same population.

If $F > F_{crit}$, the data is assumed to have come from different populations and the Department will test the remaining half of the Contractor's split samples for acceptance until the reason for the difference is resolved.

9. Calculate the pooled variance using the Contractor's and Department's variances.

$$s_p^2 = \frac{s_c^2 (n_c - 1) + s_d^2 (n_d - 1)}{n_c + n_d - 2}$$

where:

$$n_c + n_d - 2 = \text{Pooled degrees of freedom, and}$$

$$s_p^2 = \text{Pooled variance.}$$

10. Calculate the t -statistic:

$$t = \frac{|\bar{X}_c - \bar{X}_d|}{\sqrt{\frac{s_p^2}{n_c} + \frac{s_p^2}{n_d}}}$$

11. Determine the t_{crit} value from the t -distribution table for the pooled degrees of freedom ($n_c + n_d - 2$) using a significance level () of 0.01.
12. If $t \leq t_{crit}$, the data is assumed to have come from the same population and the Contractor's results will continue to be used for acceptance if the F -test results also indicate that the results have also come from the same population.

If $t > t_{crit}$, the data is assumed to have come from different populations and the Department will test the remaining half of the Contractor's split samples for acceptance until the reason for the difference is resolved.

Table 1a. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$

DEGREES OF FREEDOM FOR NUMERATOR															
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
2	199	199	199	199	199	199	199	199	199	199	199	199	199	199	
3	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4	43.3	43.2	4 3.1	
4	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7	20.6	20.5	2 0.4	
5	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4	13.3	13.2	1 3.1	
6	14. 5	12. 9	12. 0	11.5	11.1	10.8	10.6	10. 4	10.2	10.1	10. 0	9. 95	9. 88	9.81	
7	12. 4	10. 9	10. 0	9.52	9.16	8.89	8.68	8. 51	8.38	8.27	8. 18	8. 10	8. 03	7.97	
8	11. 0	9. 60	8. 81	8.30	7.95	7.69	7.50	7. 34	7.21	7.10	7. 01	6. 94	6. 87	6.81	
9	10. 1	8. 72	7. 96	7.47	7.13	6.88	6.69	6. 54	6.42	6.31	6. 23	6. 15	6. 09	6.03	
10	9. 43	8. 08	7. 34	6.87	6.54	6.30	6.12	5. 97	5.85	5.75	5. 66	5. 59	5. 53	5.47	
11	8. 91	7. 60	6. 88	6.42	6.10	5.86	5.68	5. 54	5.42	5.32	5. 24	5. 16	5. 10	5.05	
12	8. 51	7. 23	6. 52	6.07	5.76	5.52	5.35	5. 20	5.09	4.99	4. 91	4. 84	4. 77	4.72	
13	8. 19	6. 92	6. 23	5.79	5.48	5.25	5.08	4. 94	4.82	4.72	4. 64	4. 57	4. 51	4.46	
14	7. 92	6. 68	6. 00	5.56	5.26	5.03	4.86	4. 72	4.60	4.51	4. 43	4. 36	4. 30	4.25	
15	7. 70	6. 48	5. 80	5.37	5.07	4.85	4.67	4. 54	4.42	4.33	4. 25	4. 18	4. 12	4.07	
16	7. 51	6. 30	5. 64	5.21	4.91	4.69	4.52	4. 38	4.27	4.18	4. 10	4. 03	3. 97	3.92	
17	7. 35	6. 16	5. 50	5.07	4.78	4.56	4.39	4. 25	4.14	4.05	3. 97	3. 90	3. 84	3.79	
18	7. 21	6. 03	5. 37	4.96	4.66	4.44	4.28	4. 14	4.03	3.94	3. 86	3. 79	3. 73	3.68	
19	7. 09	5. 92	5. 27	4.85	4.56	4.34	4.18	4. 04	3.93	3.84	3. 76	3. 70	3. 64	3.59	
20	6. 99	5. 82	5. 17	4.76	4.47	4.26	4.09	3. 96	3.85	3.76	3. 68	3. 61	3. 55	3.50	
21	6. 91	5. 75	5. 10	4.68	4.39	4.18	4.01	3. 88	3.77	3.68	3. 60	3. 54	3. 48	3.43	
22	6. 81	5. 65	5. 02	4.61	4.32	4.11	3.94	3. 81	3.70	3.61	3. 54	3. 47	3. 41	3.36	
23	6. 73	5. 58	4. 95	4.54	4.26	4.05	3.88	3. 75	3.64	3.55	3. 47	3. 41	3. 35	3.30	
24	6. 66	5. 52	4. 89	4.49	4.20	3.99	3.83	3. 69	3.59	3.50	3. 42	3. 35	3. 30	3.25	
25	6. 60	5. 46	4. 84	4.43	4.15	3.94	3.78	3. 64	3.54	3.45	3. 37	3. 30	3. 25	3.20	

Table 1b. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR													
		2	3	4	5	6	7	8	9	10	11	12	13	14	15
DEGREES OF FREEDOM FOR DENOMINATOR	26	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49	3.40	3.33	3.26	3.20	3.15
	27	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45	3.36	3.28	3.22	3.16	3.11
	28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41	3.32	3.25	3.18	3.12	3.07
	29	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38	3.29	3.21	3.15	3.09	3.04
	30	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18	3.11	3.06	3.01
	31	6.32	5.20	4.59	4.20	3.92	3.71	3.55	3.42	3.31	3.22	3.15	3.08	3.03	2.98
	32	6.28	5.17	4.56	4.17	3.89	3.68	3.52	3.39	3.29	3.20	3.12	3.06	3.00	2.95
	33	6.25	5.14	4.53	4.14	3.86	3.66	3.49	3.37	3.26	3.17	3.09	3.03	2.97	2.92
	34	6.22	5.11	4.50	4.11	3.84	3.63	3.47	3.34	3.24	3.15	3.07	3.01	2.95	2.90
	35	6.19	5.09	4.48	4.09	3.81	3.61	3.45	3.32	3.21	3.12	3.05	2.98	2.93	2.88
	36	6.16	5.06	4.46	4.06	3.79	3.58	3.42	3.30	3.19	3.10	3.03	2.96	2.90	2.85
	37	6.13	5.04	4.43	4.04	3.77	3.56	3.40	3.28	3.17	3.08	3.01	2.94	2.88	2.83
	38	6.11	5.02	4.41	4.02	3.75	3.54	3.39	3.26	3.15	3.06	2.99	2.92	2.87	2.82
	39	6.09	5.00	4.39	4.00	3.73	3.53	3.37	3.24	3.13	3.05	2.97	2.90	2.85	2.80
	40	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95	2.89	2.83	2.78
	41	6.05	4.96	4.36	3.97	3.70	3.49	3.33	3.21	3.10	3.01	2.94	2.87	2.82	2.77
	42	6.03	4.94	4.34	3.95	3.68	3.48	3.32	3.19	3.09	3.00	2.92	2.86	2.80	2.75
	43	6.01	4.92	4.32	3.94	3.67	3.46	3.30	3.18	3.07	2.98	2.91	2.84	2.79	2.74
	44	5.99	4.91	4.31	3.92	3.65	3.45	3.29	3.16	3.06	2.97	2.89	2.83	2.77	2.72
	45	5.97	4.89	4.29	3.91	3.64	3.43	3.28	3.15	3.04	2.96	2.88	2.82	2.76	2.71
	46	5.96	4.88	4.28	3.90	3.62	3.42	3.26	3.14	3.03	2.94	2.87	2.80	2.75	2.70
	47	5.94	4.86	4.27	3.88	3.61	3.41	3.25	3.12	3.02	2.93	2.86	2.79	2.74	2.69
	48	5.93	4.85	4.25	3.87	3.60	3.40	3.24	3.11	3.01	2.92	2.85	2.78	2.72	2.67
	49	5.91	4.84	4.24	3.86	3.59	3.39	3.23	3.10	3.00	2.91	2.83	2.77	2.71	2.66
	50	5.90	4.83	4.23	3.85	3.58	3.38	3.22	3.09	2.99	2.90	2.82	2.76	2.70	2.65

Table 1c. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

DEGREES OF FREEDOM FOR NUMERATOR															
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
51	5.89	4.81	4.22	3.84	3.57	3.37	3.21	3.08	2.98	2.89	2.81	2.75	2.69	2.64	
52	5.88	4.80	4.21	3.83	3.56	3.36	3.20	3.07	2.97	2.88	2.81	2.74	2.68	2.63	
53	5.87	4.79	4.20	3.82	3.55	3.35	3.19	3.06	2.96	2.87	2.80	2.73	2.67	2.62	
54	5.85	4.78	4.19	3.81	3.54	3.34	3.18	3.05	2.95	2.86	2.79	2.72	2.67	2.62	
55	5.84	4.77	4.18	3.80	3.53	3.33	3.17	3.05	2.94	2.85	2.78	2.71	2.66	2.61	
56	5.83	4.76	4.17	3.79	3.52	3.32	3.16	3.04	2.93	2.85	2.77	2.71	2.65	2.60	
57	5.82	4.75	4.16	3.78	3.51	3.31	3.16	3.03	2.93	2.84	2.76	2.70	2.64	2.59	
58	5.81	4.75	4.16	3.78	3.51	3.31	3.15	3.02	2.92	2.83	2.76	2.69	2.63	2.58	
59	5.80	4.74	4.15	3.77	3.50	3.30	3.14	3.02	2.91	2.82	2.75	2.68	2.63	2.58	
60	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74	2.68	2.62	2.57	
61	5.79	4.72	4.13	3.75	3.48	3.28	3.13	3.00	2.90	2.81	2.74	2.67	2.61	2.56	
62	5.78	4.71	4.13	3.75	3.48	3.28	3.12	2.99	2.89	2.80	2.73	2.66	2.61	2.56	
63	5.77	4.71	4.12	3.74	3.47	3.27	3.11	2.99	2.88	2.80	2.72	2.66	2.60	2.55	
64	5.76	4.70	4.11	3.73	3.47	3.26	3.11	2.98	2.88	2.79	2.72	2.65	2.60	2.55	
65	5.75	4.69	4.11	3.73	3.46	3.26	3.10	2.98	2.87	2.79	2.71	2.65	2.59	2.54	
66	5.75	4.69	4.10	3.72	3.45	3.25	3.10	2.97	2.87	2.78	2.70	2.64	2.58	2.53	
67	5.74	4.68	4.09	3.71	3.45	3.25	3.09	2.97	2.86	2.77	2.70	2.63	2.58	2.53	
68	5.73	4.67	4.09	3.71	3.44	3.24	3.09	2.96	2.86	2.77	2.69	2.63	2.57	2.52	
69	5.73	4.67	4.08	3.70	3.44	3.24	3.08	2.95	2.85	2.76	2.69	2.62	2.57	2.52	
70	5.72	4.66	4.08	3.70	3.43	3.23	3.08	2.95	2.85	2.76	2.68	2.62	2.56	2.51	
71	5.71	4.66	4.07	3.69	3.43	3.23	3.07	2.94	2.84	2.75	2.68	2.61	2.56	2.51	
72	5.71	4.65	4.07	3.69	3.42	3.22	3.07	2.94	2.84	2.75	2.67	2.61	2.55	2.50	
73	5.70	4.64	4.06	3.68	3.42	3.22	3.06	2.94	2.83	2.74	2.67	2.61	2.55	2.50	
74	5.70	4.64	4.06	3.68	3.41	3.21	3.06	2.93	2.83	2.74	2.67	2.60	2.54	2.49	
75	5.69	4.63	4.05	3.67	3.41	3.21	3.05	2.93	2.82	2.74	2.66	2.60	2.54	2.49	

DEGREES OF FREEDOM FOR DENOMINATOR

Table 1d. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
DEGREES OF FREEDOM FOR DENOMINATOR	2	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199
	3	43.0	42.9	42.9	42.8	42.8	42.7	42.7	42.7	42.6	42.6	42.6	42.5	42.5	42.5	42.5
	4	20.4	20.3	20.3	20.2	20.2	20.1	20.1	20.1	20.0	20.0	20.0	20.0	19.9	19.9	19.9
	5	13.1	13.0	13.0	12.9	12.9	12.9	12.8	12.8	12.8	12.8	12.7	12.7	12.7	12.7	12.7
	6	9.76	9.71	9.66	9.62	9.59	9.56	9.53	9.50	9.47	9.45	9.43	9.41	9.39	9.37	9.36
	7	7.91	7.87	7.83	7.79	7.75	7.72	7.69	7.67	7.64	7.62	7.60	7.58	7.57	7.55	7.53
	8	6.76	6.72	6.68	6.64	6.61	6.58	6.55	6.53	6.50	6.48	6.46	6.44	6.43	6.41	6.40
	9	5.98	5.94	5.90	5.86	5.83	5.80	5.78	5.75	5.73	5.71	5.69	5.67	5.65	5.64	5.62
	10	5.42	5.38	5.34	5.31	5.27	5.25	5.22	5.20	5.17	5.15	5.13	5.12	5.10	5.08	5.07
	11	5.00	4.96	4.92	4.89	4.86	4.83	4.80	4.78	4.76	4.74	4.72	4.70	4.68	4.67	4.65
	12	4.67	4.63	4.59	4.56	4.53	4.50	4.48	4.45	4.43	4.41	4.39	4.38	4.36	4.34	4.33
	13	4.41	4.37	4.33	4.30	4.27	4.24	4.22	4.19	4.17	4.15	4.13	4.12	4.10	4.09	4.07
	14	4.20	4.16	4.12	4.09	4.06	4.03	4.01	3.98	3.96	3.94	3.92	3.91	3.89	3.88	3.86
	15	4.02	3.98	3.95	3.91	3.88	3.86	3.83	3.81	3.79	3.77	3.75	3.73	3.72	3.70	3.69
	16	3.87	3.83	3.80	3.76	3.73	3.71	3.68	3.66	3.64	3.62	3.60	3.58	3.57	3.55	3.54
	17	3.75	3.71	3.67	3.64	3.61	3.58	3.56	3.53	3.51	3.49	3.47	3.46	3.44	3.43	3.41
	18	3.64	3.60	3.56	3.53	3.50	3.47	3.45	3.42	3.40	3.38	3.36	3.35	3.33	3.32	3.30
	19	3.54	3.50	3.46	3.43	3.40	3.37	3.35	3.33	3.31	3.29	3.27	3.25	3.24	3.22	3.21
	20	3.46	3.42	3.38	3.35	3.32	3.29	3.27	3.24	3.22	3.20	3.18	3.17	3.15	3.14	3.12
	21	3.38	3.34	3.30	3.27	3.24	3.22	3.19	3.17	3.15	3.13	3.11	3.09	3.08	3.06	3.05
	22	3.31	3.27	3.24	3.21	3.18	3.15	3.12	3.10	3.08	3.06	3.04	3.03	3.01	3.00	2.98
	23	3.25	3.21	3.18	3.15	3.12	3.09	3.06	3.04	3.02	3.00	2.98	2.97	2.95	2.94	2.92
	24	3.20	3.16	3.12	3.09	3.06	3.04	3.01	2.99	2.97	2.95	2.93	2.91	2.90	2.88	2.87
	25	3.15	3.11	3.08	3.04	3.01	2.99	2.96	2.94	2.92	2.90	2.88	2.86	2.85	2.83	2.82

Table 1e. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
DEGREES OF FREEDOM FOR DENOMINATOR	26	3.11	3.07	3.03	3.00	2.97	2.94	2.92	2.89	2.87	2.85	2.84	2.82	2.80	2.79	2.77
	27	3.07	3.03	2.99	2.96	2.93	2.90	2.88	2.85	2.83	2.81	2.79	2.78	2.76	2.75	2.73
	28	3.03	2.99	2.95	2.92	2.89	2.86	2.84	2.82	2.79	2.77	2.76	2.74	2.72	2.71	2.69
	29	2.99	2.95	2.92	2.88	2.86	2.83	2.80	2.78	2.76	2.74	2.72	2.70	2.69	2.67	2.66
	30	2.96	2.92	2.89	2.85	2.82	2.80	2.77	2.75	2.73	2.71	2.69	2.67	2.66	2.64	2.63
	31	2.93	2.89	2.86	2.82	2.79	2.77	2.74	2.72	2.70	2.68	2.66	2.64	2.63	2.61	2.60
	32	2.90	2.86	2.83	2.80	2.77	2.74	2.71	2.69	2.67	2.65	2.63	2.61	2.60	2.58	2.57
	33	2.88	2.84	2.80	2.77	2.74	2.71	2.69	2.66	2.64	2.62	2.61	2.59	2.57	2.56	2.54
	34	2.85	2.81	2.78	2.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.56	2.55	2.53	2.52
	35	2.83	2.79	2.76	2.72	2.69	2.67	2.64	2.62	2.60	2.58	2.56	2.54	2.53	2.51	2.50
	36	2.81	2.77	2.73	2.70	2.67	2.64	2.62	2.60	2.58	2.56	2.54	2.52	2.50	2.49	2.48
	37	2.79	2.75	2.71	2.68	2.65	2.62	2.60	2.58	2.56	2.54	2.52	2.50	2.48	2.47	2.46
	38	2.77	2.73	2.70	2.66	2.63	2.61	2.58	2.56	2.54	2.52	2.50	2.48	2.47	2.45	2.44
	39	2.75	2.71	2.68	2.64	2.62	2.59	2.56	2.54	2.52	2.50	2.48	2.46	2.45	2.43	2.42
	40	2.74	2.70	2.66	2.63	2.60	2.57	2.55	2.52	2.50	2.48	2.46	2.45	2.43	2.42	2.40
	41	2.72	2.68	2.64	2.61	2.58	2.56	2.53	2.51	2.49	2.47	2.45	2.43	2.41	2.40	2.39
	42	2.71	2.67	2.63	2.60	2.57	2.54	2.52	2.49	2.47	2.45	2.43	2.42	2.40	2.38	2.37
	43	2.69	2.65	2.62	2.58	2.55	2.53	2.50	2.48	2.46	2.44	2.42	2.40	2.39	2.37	2.36
	44	2.68	2.64	2.60	2.57	2.54	2.51	2.49	2.46	2.44	2.42	2.40	2.39	2.37	2.36	2.34
	45	2.66	2.62	2.59	2.56	2.53	2.50	2.47	2.45	2.43	2.41	2.39	2.37	2.36	2.34	2.33
46	2.65	2.61	2.58	2.54	2.51	2.49	2.46	2.44	2.42	2.40	2.38	2.36	2.35	2.33	2.32	
47	2.64	2.60	2.56	2.53	2.50	2.48	2.45	2.43	2.41	2.39	2.37	2.35	2.33	2.32	2.30	
48	2.63	2.59	2.55	2.52	2.49	2.46	2.44	2.42	2.39	2.37	2.36	2.34	2.32	2.31	2.29	
49	2.62	2.58	2.54	2.51	2.48	2.45	2.43	2.40	2.38	2.36	2.35	2.33	2.31	2.30	2.28	
50	2.61	2.57	2.53	2.50	2.47	2.44	2.42	2.39	2.37	2.35	2.33	2.32	2.30	2.29	2.27	

Table 1f. Critical Values, F_{crit} for F -test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
DEGREES OF FREEDOM FOR DENOMINATOR	51	2.60	2.56	2.52	2.49	2.46	2.43	2.41	2.38	2.36	2.34	2.32	2.31	2.29	2.28	2.26
	52	2.59	2.55	2.51	2.48	2.45	2.42	2.40	2.38	2.35	2.33	2.32	2.30	2.28	2.27	2.25
	53	2.58	2.54	2.50	2.47	2.44	2.41	2.39	2.37	2.34	2.32	2.31	2.29	2.27	2.26	2.24
	54	2.57	2.53	2.50	2.46	2.43	2.41	2.38	2.36	2.34	2.32	2.30	2.28	2.26	2.25	2.23
	55	2.56	2.52	2.49	2.45	2.42	2.40	2.37	2.35	2.33	2.31	2.29	2.27	2.26	2.24	2.23
	56	2.56	2.52	2.48	2.45	2.42	2.39	2.36	2.34	2.32	2.30	2.28	2.26	2.25	2.23	2.22
	57	2.55	2.51	2.47	2.44	2.41	2.38	2.36	2.33	2.31	2.29	2.27	2.26	2.24	2.22	2.21
	58	2.54	2.50	2.46	2.43	2.40	2.37	2.35	2.33	2.30	2.28	2.27	2.25	2.23	2.22	2.20
	59	2.53	2.49	2.46	2.42	2.39	2.37	2.34	2.32	2.30	2.28	2.26	2.24	2.22	2.21	2.19
	60	2.53	2.49	2.45	2.42	2.39	2.36	2.33	2.31	2.29	2.27	2.25	2.23	2.22	2.20	2.19
	61	2.52	2.48	2.44	2.41	2.38	2.35	2.33	2.30	2.28	2.26	2.24	2.23	2.21	2.20	2.18
	62	2.51	2.47	2.44	2.40	2.37	2.35	2.32	2.30	2.28	2.26	2.24	2.22	2.20	2.19	2.17
	63	2.51	2.47	2.43	2.40	2.37	2.34	2.32	2.29	2.27	2.25	2.23	2.21	2.20	2.18	2.17
	64	2.50	2.46	2.42	2.39	2.36	2.33	2.31	2.29	2.26	2.24	2.23	2.21	2.19	2.18	2.16
	65	2.49	2.45	2.42	2.39	2.36	2.33	2.30	2.28	2.26	2.24	2.22	2.20	2.19	2.17	2.16
	66	2.49	2.45	2.41	2.38	2.35	2.32	2.30	2.27	2.25	2.23	2.21	2.20	2.18	2.16	2.15
	67	2.48	2.44	2.41	2.37	2.34	2.32	2.29	2.27	2.25	2.23	2.21	2.19	2.17	2.16	2.14
	68	2.48	2.44	2.40	2.37	2.34	2.31	2.29	2.26	2.24	2.22	2.20	2.19	2.17	2.15	2.14
	69	2.47	2.43	2.40	2.36	2.33	2.31	2.28	2.26	2.24	2.22	2.20	2.18	2.16	2.15	2.13
	70	2.47	2.43	2.39	2.36	2.33	2.30	2.28	2.25	2.23	2.21	2.19	2.17	2.16	2.14	2.13
	71	2.46	2.42	2.39	2.35	2.32	2.30	2.27	2.25	2.23	2.21	2.19	2.17	2.15	2.14	2.12
	72	2.46	2.42	2.38	2.35	2.32	2.29	2.27	2.24	2.22	2.20	2.18	2.17	2.15	2.13	2.12
	73	2.45	2.41	2.38	2.34	2.31	2.29	2.26	2.24	2.22	2.20	2.18	2.16	2.14	2.13	2.11
	74	2.45	2.41	2.37	2.34	2.31	2.28	2.26	2.23	2.21	2.19	2.17	2.16	2.14	2.12	2.11
	75	2.45	2.41	2.37	2.34	2.31	2.28	2.25	2.23	2.21	2.19	2.17	2.15	2.14	2.12	2.10

Table 1g. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
DEGREES OF FREEDOM FOR DENOMINATOR	2	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199
	3	42.5	42.4	42.4	42.4	42.4	42.4	42.4	42.3	42.3	42.3	42.3	42.3	42.3	42.3	42.3
	4	19.9	19.9	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.8	19.7	19.7	19.7	19.7	19.7
	5	12.6	12.6	12.6	12.6	12.6	12.6	12.6	12.6	12.5	12.5	12.5	12.5	12.5	12.5	12.5
	6	9.34	9.33	9.32	9.30	9.29	9.28	9.27	9.26	9.25	9.24	9.23	9.22	9.22	9.21	9.20
	7	7.52	7.51	7.49	7.48	7.47	7.46	7.45	7.44	7.43	7.42	7.41	7.41	7.40	7.39	7.38
	8	6.38	6.37	6.36	6.35	6.33	6.32	6.31	6.30	6.30	6.29	6.28	6.27	6.26	6.26	6.25
	9	5.61	5.60	5.59	5.58	5.56	5.55	5.54	5.54	5.53	5.52	5.51	5.50	5.50	5.49	5.48
	10	5.06	5.04	5.03	5.02	5.01	5.00	4.99	4.98	4.97	4.97	4.96	4.95	4.94	4.94	4.93
	11	4.64	4.63	4.62	4.61	4.60	4.59	4.58	4.57	4.56	4.55	4.54	4.54	4.53	4.52	4.52
	12	4.32	4.31	4.29	4.28	4.27	4.26	4.25	4.24	4.24	4.23	4.22	4.21	4.21	4.20	4.19
	13	4.06	4.05	4.04	4.02	4.01	4.00	4.00	3.99	3.98	3.97	3.96	3.96	3.95	3.94	3.94
	14	3.85	3.84	3.83	3.81	3.80	3.79	3.79	3.78	3.77	3.76	3.75	3.75	3.74	3.73	3.73
	15	3.67	3.66	3.65	3.64	3.63	3.62	3.61	3.60	3.59	3.58	3.58	3.57	3.56	3.56	3.55
	16	3.53	3.51	3.50	3.49	3.48	3.47	3.46	3.45	3.45	3.44	3.43	3.42	3.42	3.41	3.40
	17	3.40	3.39	3.38	3.36	3.35	3.34	3.34	3.33	3.32	3.31	3.30	3.30	3.29	3.28	3.28
	18	3.29	3.28	3.27	3.26	3.25	3.24	3.23	3.22	3.21	3.20	3.19	3.19	3.18	3.17	3.17
	19	3.19	3.18	3.17	3.16	3.15	3.14	3.13	3.12	3.11	3.11	3.10	3.09	3.08	3.08	3.07
	20	3.11	3.10	3.09	3.08	3.07	3.06	3.05	3.04	3.03	3.02	3.01	3.01	3.00	2.99	2.99
	21	2.99	2.98	2.97	2.96	2.95	2.95	2.94	2.93	2.92	2.92	2.91	2.91	2.90	2.89	2.89
	22	2.97	2.96	2.95	2.93	2.92	2.91	2.91	2.90	2.89	2.88	2.87	2.86	2.86	2.85	2.84
	23	2.91	2.90	2.89	2.87	2.86	2.85	2.85	2.84	2.83	2.82	2.81	2.80	2.80	2.79	2.78
	24	2.85	2.84	2.83	2.82	2.81	2.80	2.79	2.78	2.77	2.77	2.76	2.75	2.74	2.74	2.73
	25	2.81	2.79	2.78	2.77	2.76	2.75	2.74	2.73	2.72	2.72	2.71	2.70	2.69	2.69	2.68

Table 1h. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
DEGREES OF FREEDOM FOR DENOMINATOR	26	2.76	2.75	2.74	2.73	2.72	2.71	2.70	2.69	2.68	2.67	2.66	2.66	2.65	2.64	2.64
	27	2.72	2.71	2.70	2.68	2.67	2.66	2.66	2.65	2.64	2.63	2.62	2.61	2.61	2.60	2.59
	28	2.68	2.67	2.66	2.65	2.64	2.63	2.62	2.61	2.60	2.59	2.58	2.58	2.57	2.56	2.56
	29	2.65	2.63	2.62	2.61	2.60	2.59	2.58	2.57	2.56	2.56	2.55	2.54	2.53	2.53	2.52
	30	2.61	2.60	2.59	2.58	2.57	2.56	2.55	2.54	2.53	2.52	2.52	2.51	2.50	2.49	2.49
	31	2.58	2.57	2.56	2.55	2.54	2.53	2.52	2.51	2.50	2.49	2.49	2.48	2.47	2.46	2.46
	32	2.56	2.54	2.53	2.52	2.51	2.50	2.49	2.48	2.47	2.47	2.46	2.45	2.44	2.44	2.43
	33	2.53	2.52	2.51	2.50	2.48	2.47	2.47	2.46	2.45	2.44	2.43	2.42	2.42	2.41	2.40
	34	2.51	2.49	2.48	2.47	2.46	2.45	2.44	2.43	2.42	2.42	2.41	2.40	2.39	2.39	2.38
	35	2.48	2.47	2.46	2.45	2.44	2.43	2.42	2.41	2.40	2.39	2.38	2.38	2.37	2.36	2.36
	36	2.46	2.45	2.44	2.43	2.42	2.41	2.40	2.39	2.38	2.37	2.36	2.36	2.35	2.34	2.33
	37	2.44	2.43	2.42	2.41	2.40	2.39	2.38	2.37	2.36	2.35	2.34	2.33	2.33	2.32	2.31
	38	2.42	2.41	2.40	2.39	2.38	2.37	2.36	2.35	2.34	2.33	2.32	2.32	2.31	2.30	2.29
	39	2.41	2.39	2.38	2.37	2.36	2.35	2.34	2.33	2.32	2.31	2.31	2.30	2.29	2.28	2.28
	40	2.39	2.38	2.36	2.35	2.34	2.33	2.32	2.31	2.30	2.30	2.29	2.28	2.27	2.27	2.26
	41	2.37	2.36	2.35	2.34	2.33	2.32	2.31	2.30	2.29	2.28	2.27	2.26	2.26	2.25	2.24
	42	2.36	2.34	2.33	2.32	2.31	2.30	2.29	2.28	2.27	2.26	2.26	2.25	2.24	2.23	2.23
	43	2.34	2.33	2.32	2.31	2.30	2.29	2.28	2.27	2.26	2.25	2.24	2.23	2.23	2.22	2.21
	44	2.33	2.32	2.30	2.29	2.28	2.27	2.26	2.25	2.24	2.24	2.23	2.22	2.21	2.21	2.20
	45	2.32	2.30	2.29	2.28	2.27	2.26	2.25	2.24	2.23	2.22	2.21	2.21	2.20	2.19	2.19
46	2.30	2.29	2.28	2.27	2.26	2.25	2.24	2.23	2.22	2.21	2.20	2.19	2.19	2.18	2.17	
47	2.29	2.28	2.27	2.25	2.24	2.23	2.22	2.21	2.21	2.20	2.19	2.18	2.17	2.17	2.16	
48	2.28	2.27	2.25	2.24	2.23	2.22	2.21	2.20	2.19	2.19	2.18	2.17	2.16	2.16	2.15	
49	2.27	2.26	2.24	2.23	2.22	2.21	2.20	2.19	2.18	2.18	2.17	2.16	2.15	2.14	2.14	
50	2.26	2.25	2.23	2.22	2.21	2.20	2.19	2.18	2.17	2.16	2.16	2.15	2.14	2.13	2.13	

Table 1i. Critical Values, F_{crit} , for F -test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
DEGREES OF FREEDOM FOR DENOMINATOR	51	2.25	2.24	2.22	2.21	2.20	2.19	2.18	2.17	2.16	2.15	2.15	2.14	2.13	2.12	2.12
	52	2.24	2.23	2.21	2.20	2.19	2.18	2.17	2.16	2.15	2.14	2.14	2.13	2.12	2.11	2.11
	53	2.23	2.22	2.20	2.19	2.18	2.17	2.16	2.15	2.14	2.14	2.13	2.12	2.11	2.10	2.10
	54	2.22	2.21	2.20	2.18	2.17	2.16	2.15	2.14	2.13	2.13	2.12	2.11	2.10	2.10	2.09
	55	2.21	2.20	2.19	2.18	2.16	2.15	2.14	2.14	2.13	2.12	2.11	2.10	2.09	2.09	2.08
	56	2.20	2.19	2.18	2.17	2.16	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.09	2.08	2.07
	57	2.20	2.18	2.17	2.16	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.09	2.08	2.07	2.06
	58	2.19	2.18	2.16	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.09	2.08	2.07	2.06	2.06
	59	2.18	2.17	2.16	2.14	2.13	2.12	2.11	2.10	2.09	2.09	2.08	2.07	2.06	2.06	2.05
	60	2.17	2.16	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.06	2.05	2.04
	61	2.17	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.06	2.05	2.04	2.03
	62	2.16	2.15	2.14	2.12	2.11	2.10	2.09	2.08	2.07	2.07	2.06	2.05	2.04	2.03	2.03
	63	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.03	2.02
	64	2.15	2.13	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.04	2.03	2.02	2.01
	65	2.14	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06	2.05	2.04	2.03	2.02	2.02	2.01
	66	2.14	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.02	2.01	2.00
	67	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.03	2.02	2.01	2.00	2.00
	68	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.01	2.00	1.99
	69	2.12	2.11	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.02	2.01	2.00	1.99	1.99
	70	2.11	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.00	1.99	1.99	1.98
	71	2.11	2.10	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.01	2.00	1.99	1.98	1.98
	72	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.00	1.99	1.98	1.98	1.97
	73	2.10	2.09	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.00	2.00	1.99	1.98	1.97	1.97
	74	2.10	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.00	1.99	1.98	1.98	1.97	1.96
	75	2.09	2.08	2.07	2.05	2.04	2.03	2.02	2.01	2.00	1.99	1.99	1.98	1.97	1.96	1.96

Table 1j. Critical Values, F_{crit} for F -test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
DEGREES OF FREEDOM FOR DENOMINATOR	2	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199
	3	42.3	42.2	42.2	42.2	42.2	42.2	42.2	42.2	42.2	42.2	42.2	42.2	42.2	42.2	42.2
	4	19.7	19.7	19.7	19.7	19.7	19.7	19.7	19.7	19.6	19.6	19.6	19.6	19.6	19.6	19.6
	5	12.5	12.5	12.5	12.5	12.5	12.5	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4
	6	9.19	9.19	9.18	9.18	9.17	9.16	9.16	9.15	9.15	9.14	9.14	9.13	9.13	9.13	9.12
	7	7.38	7.37	7.37	7.36	7.35	7.35	7.34	7.34	7.33	7.33	7.33	7.32	7.32	7.31	7.31
	8	6.24	6.24	6.23	6.23	6.22	6.22	6.21	6.21	6.20	6.20	6.19	6.19	6.18	6.18	6.18
	9	5.48	5.47	5.46	5.46	5.45	5.45	5.44	5.44	5.43	5.43	5.43	5.42	5.42	5.41	5.41
	10	4.92	4.92	4.91	4.91	4.90	4.90	4.89	4.89	4.88	4.88	4.87	4.87	4.87	4.86	4.86
	11	4.51	4.50	4.50	4.49	4.49	4.48	4.48	4.47	4.47	4.46	4.46	4.46	4.45	4.45	4.45
	12	4.19	4.18	4.18	4.17	4.17	4.16	4.16	4.15	4.15	4.14	4.14	4.13	4.13	4.13	4.12
	13	3.93	3.92	3.92	3.91	3.91	3.90	3.90	3.89	3.89	3.88	3.88	3.88	3.87	3.87	3.87
	14	3.72	3.71	3.71	3.70	3.70	3.69	3.69	3.68	3.68	3.67	3.67	3.67	3.66	3.66	3.66
	15	3.54	3.54	3.53	3.53	3.52	3.52	3.51	3.51	3.50	3.50	3.50	3.49	3.49	3.48	3.48
	16	3.40	3.39	3.39	3.38	3.37	3.37	3.37	3.36	3.36	3.35	3.35	3.34	3.34	3.34	3.33
	17	3.27	3.26	3.26	3.25	3.25	3.24	3.24	3.23	3.23	3.23	3.22	3.22	3.21	3.21	3.21
	18	3.16	3.15	3.15	3.14	3.14	3.13	3.13	3.12	3.12	3.12	3.11	3.11	3.10	3.10	3.10
	19	3.06	3.06	3.05	3.05	3.04	3.04	3.03	3.03	3.02	3.02	3.02	3.01	3.01	3.00	3.00
	20	2.98	2.97	2.97	2.96	2.96	2.95	2.95	2.94	2.94	2.94	2.93	2.93	2.92	2.92	2.92
	21	2.91	2.90	2.90	2.89	2.88	2.88	2.87	2.87	2.86	2.86	2.86	2.85	2.85	2.84	2.84
	22	2.84	2.83	2.83	2.82	2.82	2.81	2.81	2.80	2.80	2.79	2.79	2.79	2.78	2.78	2.77
	23	2.78	2.77	2.77	2.76	2.76	2.75	2.75	2.74	2.74	2.73	2.73	2.72	2.72	2.72	2.71
	24	2.72	2.72	2.71	2.71	2.70	2.70	2.69	2.69	2.68	2.68	2.67	2.67	2.67	2.66	2.66
	25	2.67	2.67	2.66	2.66	2.65	2.65	2.64	2.64	2.63	2.63	2.62	2.62	2.62	2.61	2.61

Table 1k. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR															
		46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	
DEGREES OF FREEDOM FOR DENOMINATOR	26	2.63	2.62	2.62	2.61	2.61	2.60	2.60	2.59	2.59	2.58	2.58	2.57	2.57	2.57	2.56	
	27	2.59	2.58	2.58	2.57	2.57	2.56	2.56	2.55	2.55	2.54	2.54	2.53	2.53	2.53	2.52	
	28	2.55	2.54	2.54	2.53	2.53	2.52	2.52	2.51	2.51	2.50	2.50	2.50	2.49	2.49	2.48	
	29	2.51	2.51	2.50	2.50	2.49	2.49	2.48	2.48	2.47	2.47	2.46	2.46	2.46	2.45	2.45	
	30	2.48	2.48	2.47	2.46	2.46	2.45	2.45	2.44	2.44	2.44	2.43	2.43	2.42	2.42	2.42	
	31	2.45	2.45	2.44	2.43	2.43	2.42	2.42	2.41	2.41	2.40	2.40	2.40	2.39	2.39	2.38	
	32	2.42	2.42	2.41	2.41	2.40	2.40	2.39	2.39	2.38	2.38	2.37	2.37	2.36	2.36	2.36	
	33	2.40	2.39	2.39	2.38	2.37	2.37	2.36	2.36	2.35	2.35	2.35	2.34	2.34	2.33	2.33	
	34	2.37	2.37	2.36	2.36	2.35	2.34	2.34	2.33	2.33	2.33	2.32	2.32	2.31	2.31	2.30	
	35	2.35	2.34	2.34	2.33	2.33	2.32	2.32	2.31	2.31	2.30	2.30	2.29	2.29	2.29	2.28	
	36	2.33	2.32	2.32	2.31	2.30	2.30	2.29	2.29	2.28	2.28	2.28	2.27	2.27	2.26	2.26	
	37	2.31	2.30	2.30	2.29	2.28	2.28	2.27	2.27	2.26	2.26	2.26	2.25	2.25	2.24	2.24	
	38	2.29	2.28	2.28	2.27	2.27	2.26	2.25	2.25	2.25	2.24	2.24	2.23	2.23	2.22	2.22	
	39	2.27	2.26	2.26	2.25	2.25	2.24	2.24	2.23	2.23	2.22	2.22	2.21	2.21	2.21	2.20	
	40	2.25	2.25	2.24	2.24	2.23	2.22	2.22	2.21	2.21	2.20	2.20	2.20	2.19	2.19	2.18	
	41	2.24	2.23	2.22	2.22	2.21	2.21	2.20	2.20	2.19	2.19	2.18	2.18	2.18	2.17	2.17	
	42	2.22	2.21	2.21	2.20	2.20	2.19	2.19	2.18	2.18	2.17	2.17	2.16	2.16	2.16	2.15	
	43	2.21	2.20	2.19	2.19	2.18	2.18	2.17	2.17	2.16	2.16	2.15	2.15	2.14	2.14	2.14	
	44	2.19	2.19	2.18	2.17	2.17	2.16	2.16	2.15	2.15	2.14	2.14	2.13	2.13	2.13	2.12	
	45	2.18	2.17	2.17	2.16	2.16	2.15	2.14	2.14	2.13	2.13	2.13	2.12	2.12	2.11	2.11	
46	2.17	2.16	2.15	2.15	2.14	2.14	2.13	2.13	2.12	2.12	2.11	2.11	2.10	2.10	2.10		
47	2.15	2.15	2.14	2.14	2.13	2.12	2.12	2.11	2.11	2.11	2.10	2.10	2.09	2.09	2.08		
48	2.14	2.14	2.13	2.12	2.12	2.11	2.11	2.10	2.10	2.09	2.09	2.08	2.08	2.08	2.07		
49	2.13	2.12	2.12	2.11	2.11	2.10	2.10	2.09	2.09	2.08	2.08	2.07	2.07	2.06	2.06		
50	2.12	2.11	2.11	2.10	2.10	2.09	2.09	2.08	2.08	2.07	2.07	2.06	2.06	2.05	2.05		

Table 11. Critical Values, F_{crit} , for F -test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

DEGREES OF FREEDOM FOR NUMERATOR																
		46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
	51	2.11	2.10	2.10	2.09	2.09	2.08	2.08	2.07	2.07	2.06	2.06	2.05	2.05	2.04	2.04
	52	2.10	2.09	2.09	2.08	2.08	2.07	2.07	2.06	2.06	2.05	2.05	2.04	2.04	2.03	2.03
	53	2.09	2.08	2.08	2.07	2.07	2.06	2.06	2.05	2.05	2.04	2.04	2.03	2.03	2.02	2.02
	54	2.08	2.08	2.07	2.06	2.06	2.05	2.05	2.04	2.04	2.03	2.03	2.02	2.02	2.01	2.01
	55	2.07	2.07	2.06	2.05	2.05	2.04	2.04	2.03	2.03	2.02	2.02	2.01	2.01	2.01	2.00
	56	2.06	2.06	2.05	2.05	2.04	2.04	2.03	2.02	2.02	2.02	2.01	2.01	2.00	2.00	1.99
	57	2.06	2.05	2.04	2.04	2.03	2.03	2.02	2.02	2.01	2.01	2.00	2.00	1.99	1.99	1.99
	58	2.05	2.04	2.04	2.03	2.02	2.02	2.01	2.01	2.00	2.00	1.99	1.99	1.99	1.98	1.98
	59	2.04	2.04	2.03	2.02	2.02	2.01	2.01	2.00	2.00	1.99	1.99	1.98	1.98	1.97	1.97
	60	2.03	2.03	2.02	2.02	2.01	2.00	2.00	1.99	1.99	1.98	1.98	1.97	1.97	1.97	1.96
	61	2.03	2.02	2.01	2.01	2.00	2.00	1.99	1.99	1.98	1.98	1.97	1.97	1.96	1.96	1.96
	62	2.02	2.01	2.01	2.00	2.00	1.99	1.99	1.98	1.97	1.97	1.97	1.96	1.96	1.95	1.95
	63	2.01	2.01	2.00	2.00	1.99	1.98	1.98	1.97	1.97	1.96	1.96	1.95	1.95	1.95	1.94
	64	2.01	2.00	1.99	1.99	1.98	1.98	1.97	1.97	1.96	1.96	1.95	1.95	1.94	1.94	1.93
	65	2.00	2.00	1.99	1.98	1.98	1.97	1.97	1.96	1.96	1.95	1.95	1.94	1.94	1.93	1.93
	66	2.00	1.99	1.98	1.98	1.97	1.97	1.96	1.95	1.95	1.94	1.94	1.94	1.93	1.93	1.92
	67	1.99	1.98	1.98	1.97	1.97	1.96	1.95	1.95	1.94	1.94	1.93	1.93	1.93	1.92	1.92
	68	1.98	1.98	1.97	1.97	1.96	1.95	1.95	1.94	1.94	1.93	1.93	1.92	1.92	1.92	1.91
	69	1.98	1.97	1.97	1.96	1.95	1.95	1.94	1.94	1.93	1.93	1.92	1.92	1.91	1.91	1.91
	70	1.97	1.97	1.96	1.95	1.95	1.94	1.94	1.93	1.93	1.92	1.92	1.91	1.91	1.90	1.90
	71	1.97	1.96	1.96	1.95	1.94	1.94	1.93	1.93	1.92	1.92	1.91	1.91	1.90	1.90	1.89
	72	1.96	1.96	1.95	1.94	1.94	1.93	1.93	1.92	1.92	1.91	1.91	1.90	1.90	1.89	1.89
	73	1.96	1.95	1.95	1.94	1.93	1.93	1.92	1.92	1.91	1.91	1.90	1.90	1.89	1.89	1.88
	74	1.95	1.95	1.94	1.93	1.93	1.92	1.92	1.91	1.91	1.90	1.90	1.89	1.89	1.88	1.88
	75	1.95	1.94	1.94	1.93	1.92	1.92	1.91	1.91	1.90	1.90	1.89	1.89	1.88	1.88	1.88

DEGREES OF FREEDOM FOR DENOMINATOR

Table 1m. Critical Values, F_{crit} for F -test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
DEGREES OF FREEDOM FOR DENOMINATOR	2	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199
	3	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1	42.1
	4	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6	19.6
	5	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4	12.4
	6	9.12	9.11	9.11	9.11	9.10	9.10	9.10	9.09	9.09	9.09	9.08	9.08	9.08	9.08	9.07
	7	7.30	7.30	7.30	7.29	7.29	7.29	7.28	7.28	7.28	7.28	7.27	7.27	7.27	7.27	7.26
	8	6.17	6.17	6.17	6.16	6.16	6.16	6.15	6.15	6.15	6.15	6.14	6.14	6.14	6.13	6.13
	9	5.41	5.40	5.40	5.40	5.39	5.39	5.39	5.38	5.38	5.38	5.38	5.37	5.37	5.37	5.37
	10	4.86	4.85	4.85	4.85	4.84	4.84	4.84	4.83	4.83	4.83	4.83	4.82	4.82	4.82	4.82
	11	4.44	4.44	4.43	4.43	4.43	4.43	4.42	4.42	4.42	4.41	4.41	4.41	4.41	4.40	4.40
	12	4.12	4.12	4.11	4.11	4.11	4.10	4.10	4.10	4.10	4.09	4.09	4.09	4.08	4.08	4.08
	13	3.86	3.86	3.86	3.85	3.85	3.85	3.84	3.84	3.84	3.84	3.83	3.83	3.83	3.83	3.82
	14	3.65	3.65	3.65	3.64	3.64	3.64	3.63	3.63	3.63	3.62	3.62	3.62	3.62	3.61	3.61
	15	3.48	3.47	3.47	3.47	3.46	3.46	3.46	3.46	3.45	3.45	3.45	3.44	3.44	3.44	3.44
	16	3.33	3.33	3.32	3.32	3.32	3.31	3.31	3.31	3.30	3.30	3.30	3.30	3.29	3.29	3.29
	17	3.20	3.20	3.20	3.19	3.19	3.19	3.18	3.18	3.18	3.18	3.17	3.17	3.17	3.17	3.16
	18	3.09	3.09	3.09	3.08	3.08	3.08	3.07	3.07	3.07	3.07	3.06	3.06	3.06	3.06	3.05
	19	3.00	2.99	2.99	2.99	2.98	2.98	2.98	2.98	2.97	2.97	2.97	2.96	2.96	2.96	2.96
	20	2.91	2.91	2.91	2.90	2.90	2.90	2.89	2.89	2.89	2.88	2.88	2.88	2.88	2.87	2.87
	21	2.84	2.84	2.83	2.83	2.82	2.82	2.82	2.82	2.81	2.81	2.81	2.80	2.80	2.80	2.80
	22	2.77	2.77	2.76	2.76	2.76	2.75	2.75	2.75	2.75	2.74	2.74	2.74	2.73	2.73	2.73
	23	2.71	2.71	2.70	2.70	2.70	2.69	2.69	2.69	2.68	2.68	2.68	2.68	2.67	2.67	2.67
	24	2.65	2.65	2.65	2.64	2.64	2.64	2.64	2.63	2.63	2.63	2.62	2.62	2.62	2.62	2.61
	25	2.61	2.60	2.60	2.59	2.59	2.59	2.59	2.58	2.58	2.58	2.57	2.57	2.57	2.57	2.56

Table 1n. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR														
		61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
DEGREES OF FREEDOM FOR DENOMINATOR	26	2.56	2.56	2.55	2.55	2.55	2.54	2.54	2.54	2.53	2.53	2.53	2.53	2.52	2.52	2.52
	27	2.52	2.51	2.51	2.51	2.50	2.50	2.50	2.50	2.49	2.49	2.49	2.48	2.48	2.48	2.48
	28	2.48	2.48	2.47	2.47	2.47	2.46	2.46	2.46	2.45	2.45	2.45	2.45	2.44	2.44	2.44
	29	2.44	2.44	2.44	2.43	2.43	2.43	2.42	2.42	2.42	2.42	2.41	2.41	2.41	2.41	2.40
	30	2.41	2.41	2.40	2.40	2.40	2.39	2.39	2.39	2.39	2.38	2.38	2.38	2.37	2.37	2.37
	31	2.38	2.38	2.37	2.37	2.37	2.36	2.36	2.36	2.36	2.35	2.35	2.35	2.34	2.34	2.34
	32	2.35	2.35	2.35	2.34	2.34	2.34	2.33	2.33	2.33	2.32	2.32	2.32	2.32	2.31	2.31
	33	2.33	2.32	2.32	2.32	2.31	2.31	2.31	2.30	2.30	2.30	2.29	2.29	2.29	2.29	2.28
	34	2.30	2.30	2.29	2.29	2.29	2.28	2.28	2.28	2.28	2.27	2.27	2.27	2.26	2.26	2.26
	35	2.28	2.27	2.27	2.27	2.26	2.26	2.26	2.25	2.25	2.25	2.25	2.24	2.24	2.24	2.24
	36	2.26	2.25	2.25	2.25	2.24	2.24	2.24	2.23	2.23	2.23	2.22	2.22	2.22	2.22	2.21
	37	2.24	2.23	2.23	2.22	2.22	2.22	2.21	2.21	2.21	2.21	2.20	2.20	2.20	2.20	2.19
	38	2.22	2.21	2.21	2.21	2.20	2.20	2.20	2.19	2.19	2.19	2.18	2.18	2.18	2.18	2.17
	39	2.20	2.19	2.19	2.19	2.18	2.18	2.18	2.17	2.17	2.17	2.17	2.16	2.16	2.16	2.15
	40	2.18	2.18	2.17	2.17	2.17	2.16	2.16	2.16	2.15	2.15	2.15	2.14	2.14	2.14	2.14
	41	2.16	2.16	2.16	2.15	2.15	2.15	2.14	2.14	2.14	2.13	2.13	2.13	2.13	2.12	2.12
	42	2.15	2.14	2.14	2.14	2.13	2.13	2.13	2.12	2.12	2.12	2.12	2.11	2.11	2.11	2.10
	43	2.13	2.13	2.13	2.12	2.12	2.12	2.11	2.11	2.11	2.10	2.10	2.10	2.09	2.09	2.09
	44	2.12	2.11	2.11	2.11	2.10	2.10	2.10	2.09	2.09	2.09	2.09	2.08	2.08	2.08	2.07
	45	2.11	2.10	2.10	2.09	2.09	2.09	2.08	2.08	2.08	2.08	2.07	2.07	2.07	2.06	2.06
46	2.09	2.09	2.08	2.08	2.08	2.07	2.07	2.07	2.07	2.06	2.06	2.06	2.05	2.05	2.05	
47	2.08	2.08	2.07	2.07	2.07	2.06	2.06	2.06	2.05	2.05	2.05	2.04	2.04	2.04	2.04	
48	2.07	2.06	2.06	2.06	2.05	2.05	2.05	2.04	2.04	2.04	2.03	2.03	2.03	2.03	2.02	
49	2.06	2.05	2.05	2.05	2.04	2.04	2.04	2.03	2.03	2.03	2.02	2.02	2.02	2.02	2.01	
50	2.05	2.04	2.04	2.03	2.03	2.03	2.02	2.02	2.02	2.02	2.01	2.01	2.01	2.00	2.00	

Table 1o. Critical Values, F_{crit} , for F-test using two-tailed distribution for a Level of Significance, $\alpha = 0.01$ (continued)

		DEGREES OF FREEDOM FOR NUMERATOR															
		61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	
DEGREES OF FREEDOM FOR DENOMINATOR	51	2.04	2.03	2.03	2.02	2.02	2.02	2.01	2.01	2.01	2.00	2.00	2.00	2.00	1.99	1.99	
	52	2.03	2.02	2.02	2.01	2.01	2.01	2.00	2.00	2.00	1.99	1.99	1.99	1.99	1.98	1.98	
	53	2.02	2.01	2.01	2.00	2.00	2.00	1.99	1.99	1.99	1.99	1.98	1.98	1.98	1.97	1.97	
	54	2.01	2.00	2.00	2.00	1.99	1.99	1.99	1.98	1.98	1.98	1.97	1.97	1.97	1.96	1.96	
	55	2.00	1.99	1.99	1.99	1.98	1.98	1.98	1.97	1.97	1.97	1.96	1.96	1.96	1.96	1.95	
	56	1.99	1.99	1.98	1.98	1.97	1.97	1.97	1.96	1.96	1.96	1.96	1.96	1.95	1.95	1.95	1.94
	57	1.98	1.98	1.97	1.97	1.97	1.96	1.96	1.96	1.95	1.95	1.95	1.95	1.94	1.94	1.94	1.94
	58	1.97	1.97	1.97	1.96	1.96	1.95	1.95	1.95	1.95	1.94	1.94	1.94	1.94	1.93	1.93	1.93
	59	1.97	1.96	1.96	1.95	1.95	1.95	1.94	1.94	1.94	1.93	1.93	1.93	1.93	1.93	1.92	1.92
	60	1.96	1.95	1.95	1.95	1.94	1.94	1.94	1.93	1.93	1.93	1.92	1.92	1.92	1.92	1.92	1.91
	61	1.95	1.95	1.94	1.94	1.94	1.93	1.93	1.93	1.92	1.92	1.92	1.92	1.91	1.91	1.91	1.91
	62	1.94	1.94	1.94	1.93	1.93	1.93	1.92	1.92	1.92	1.91	1.91	1.91	1.91	1.90	1.90	1.90
	63	1.94	1.93	1.93	1.93	1.92	1.92	1.92	1.91	1.91	1.91	1.91	1.90	1.90	1.90	1.89	1.89
	64	1.93	1.93	1.92	1.92	1.92	1.91	1.91	1.91	1.90	1.90	1.90	1.90	1.89	1.89	1.89	1.88
	65	1.92	1.92	1.92	1.91	1.91	1.91	1.90	1.90	1.90	1.89	1.89	1.89	1.88	1.88	1.88	1.88
	66	1.92	1.91	1.91	1.91	1.90	1.90	1.90	1.89	1.89	1.89	1.88	1.88	1.88	1.88	1.88	1.87
	67	1.91	1.91	1.90	1.90	1.90	1.89	1.89	1.89	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87
	68	1.91	1.90	1.90	1.90	1.89	1.89	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.86	1.86
	69	1.90	1.90	1.89	1.89	1.89	1.88	1.88	1.88	1.87	1.87	1.87	1.86	1.86	1.86	1.86	1.86
	70	1.90	1.89	1.89	1.88	1.88	1.88	1.87	1.87	1.87	1.86	1.86	1.86	1.86	1.86	1.85	1.85
	71	1.89	1.89	1.88	1.88	1.88	1.87	1.87	1.87	1.86	1.86	1.86	1.85	1.85	1.85	1.85	1.84
	72	1.89	1.88	1.88	1.87	1.87	1.87	1.86	1.86	1.86	1.85	1.85	1.85	1.84	1.84	1.84	1.84
	73	1.88	1.88	1.87	1.87	1.87	1.86	1.86	1.86	1.85	1.85	1.85	1.84	1.84	1.84	1.84	1.83
	74	1.88	1.87	1.87	1.86	1.86	1.86	1.85	1.85	1.85	1.84	1.84	1.84	1.83	1.83	1.83	1.83
	75	1.87	1.87	1.86	1.86	1.86	1.85	1.85	1.85	1.84	1.84	1.84	1.83	1.83	1.83	1.83	1.82

Table 2a. Critical Values, t_{crit} , using two-tailed distribution for the t -test for a Level of Significance, $\alpha = 0.01$

Critical Values, t_{crit} for the t -test			
Degrees of Freedom	$\alpha = 0.01$	Degrees of Freedom	$\alpha = 0.01$
1	63.657	21	2.831
2	9.925	22	2.819
3	5.841	23	2.807
4	4.604	24	2.797
5	4.032	25	2.787
6	3.707	26	2.779
7	3.499	27	2.771
8	3.355	28	2.763
9	3.250	29	2.756
10	3.169	30	2.750
11	3.106	31	2.744
12	3.055	32	2.738
13	3.012	33	2.733
14	2.977	34	2.728
15	2.947	35	2.723
16	2.921	36	2.719
17	2.898	37	2.715
18	2.878	38	2.711
19	2.861	39	2.707
20	2.845	40	2.704

Table 2b. Critical Values, t_{crit} , using two-tailed distribution for the t -test for a Level of Significance, $\alpha = 0.01$ (continued)

Critical Values, t_{crit} for the t -test			
Degrees of Freedom	$\alpha = 0.01$	Degrees of Freedom	$\alpha = 0.01$
41	2.701	71	2.647
42	2.698	72	2.646
43	2.695	73	2.645
44	2.692	74	2.644
45	2.690	75	2.643
46	2.687	76	2.642
47	2.685	77	2.641
48	2.682	78	2.640
49	2.680	79	2.640
50	2.678	80	2.639
51	2.676	81	2.638
52	2.674	82	2.637
53	2.672	83	2.636
54	2.670	84	2.636
55	2.668	85	2.635
56	2.667	86	2.634
57	2.665	87	2.634
58	2.663	88	2.633
59	2.662	89	2.632
60	2.660	90	2.632
61	2.659	91	2.632
62	2.657	92	2.631
63	2.656	93	2.630
64	2.655	94	2.630
65	2.654	95	2.629
66	2.652	96	2.629
67	2.651	97	2.628
68	2.650	98	2.627
69	2.649	99	2.627
70	2.648	100	2.626