

## Appendix A

In order to utilize the Contractor's material test results for acceptance and payment, the Department must ensure that the Contractor's results compare favorably with the Department's test results for the same lot of material. This comparison for asphalt is made by use of the latest version of the CCD 200215 Excel file located on the Materials & Research Division's Common Asphalt Files web site: [http://www.ok.gov/odot/Doing\\_Business/Construction/Materials\\_&\\_Testing\\_e-Guide/comm\\_asphss.html](http://www.ok.gov/odot/Doing_Business/Construction/Materials_&_Testing_e-Guide/comm_asphss.html)

Statistical methods utilized in the Excel file and its rounding will govern. In general, computations will be shown to three decimal places though not typically rounded in the Excel file. For the examples detailed herein, the numbers will be rounded to make intermediate steps more easily shown in the equations and tables in Appendix B-D. Refer to Appendix B for generic equation references in the sample data examples. Numbers in brackets [ ]'s, are values computed by the Excel file.

A minimum of four test sets are required for the first comparison. This initialization comparison for acceptance will use paired split t-tests to determine compliance and acceptance. Ongoing comparisons will use F&t independent test comparisons.

Sample AC data:

Set	Lot / Sub-Lot	AC $X_d$	AC $X_c$
1	1/1	4.1	4.2
2	1/2	4.2	4.4
3	1/3	4.1	4.3
4	1/4	4.2	4.2

While lot/sub-lot paired split samples as listed are typically used for QC/QA and PWL specifications, four standard 1,000 ton lots can be used as well. Due to number of samples and therefore quantities needed for comparison, projects with QC/QA and PWL Special Provisions will be the best projects to consider as candidate projects to warrant the effort required to utilize this method.

### Initialization – Equations 1-4

*Equation 1 – Compute the mean (average) of the difference between paired tests:*

1. For each of the paired samples, compute the numerator first.
  - a. Sum the difference for each Contractor test result for AC minus the Department's test result for AC.
2. Divide this sum by the number of paired sets which is 4 in this case.

$$\bar{X}_p = \frac{0.500}{4} \cong 0.125$$

*Equation 2 – Compute the standard deviation of the difference between paired tests:*

1. For each of the paired samples, compute the numerator first.

- a. Sum the difference for each Contractor test result for AC minus the Department’s test result for AC minus the paired mean computed in equation 1.
2. Divide this sum by the number of paired sets minus 1 to determine the denominator which is the degrees of freedom, 3 in this case.
3. Take the square root of the result or raise that number to the 0.5 power.

$$S_p = \sqrt{\frac{0.028}{4 - 1}} \cong 0.096$$

Equation 3 – Compute the paired t-test statistic:

1. Substitute the values computed previously into equation 3 and solve.

$$t_p = \left| \sqrt{4} \frac{0.125}{0.096} \right| \cong 2.611 [2.577]$$

Equation 4 – Compute the critical or maximum allowable t-test result:

1. Compute the degrees of freedom for paired tests.

$$D_f = n_p - 1 = 4 - 1 = 3$$

2. Compute t-critical:
  - a. Compute t-critical using the Excel formula for a two-tailed Student’s t-distribution with a probability of 0.01.

$$t_{crit} = T.INV.2T(\alpha, D_f) = T.INV.2T(0.01, 3) \cong 5.841$$

- b. For estimation purposes, use table 1 in Appendix C to lookup t-critical for this case where degrees of freedom is 3.

Critical Values of the t-statistic two tailed $\alpha = 0.01$							
$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$
2	9.925	14	2.977	26	2.779	38	2.712
3	5.841	15	2.947	27	2.771	39	2.708

Accept Contractor’s Test Results Based on Initialization Results:

1. If the paired t-statistic is less than or equal to t-critical, accept.

$$t_p \leq t_{crit} \qquad 2.611 [2.577] \leq 5.841 = \text{TRUE}$$

OnGoing – Equations 5-15

Before the ongoing acceptance computations using F&t statistics can begin, the mixture sample sets that are compared must be selected. These sets must be stratified randomly selected and independent. Use the Random worksheet in the CCD 2000215 Excel file to pick random sets should the Residency (Department) lab tests a split sample where the Contractor tests the other half of the split sample. It is critical that both test results are not selected as a set selected for this comparison. The Residency may also choose to not split samples with the Contractor for their ongoing stratified random test sets. As a third option, some samples could be split and independent for some sets and some that are not split. Roadway core tests are the exception where the Residency or Contractor may choose to share the cores if non-destructively tested by the Residency first.

A minimum of 25% of the tests must be tested by the Residency. This means that for the average PWL lot size of 4 sublots, 4,000 tons, then one of the four 1,000 ton sublots are randomly selected. As explained earlier, this mixture sample can be split with the Contractor and not use the Contractor's split sample result or choose to randomly sample and split the sample fully independent or mix the two methods of random selection and splitting option.

The Residency should not inform the Contractor which of the four sets of tests will be picked and tested until tests are completed for the ongoing comparisons and all four sets are tested by the Contractor. Once the Residency receives all four test results, they should share their results with the Contractor within 48 working hours. In the case of the initialization paired tests, share each test set within 48 working hours after receiving the sample.

A minimum of three Residency test sets are required for the first ongoing analysis. After initialization, randomly pick two of those four test sets for the Residency and select the other two for the Contractor test sets. For the first ongoing comparison and others, for the Residency set of tests, add the two initialization test sets to one of four for the next set of four sublots for a total of two lots in the case of PWL for the first ongoing comparison.

The random test sets and independent requirements for the ongoing comparisons for acceptance as explained above may seem complicated. The sample AC data after the paragraph below shows how the independent and split method works which may be more easily understood for the first ongoing comparison. While you can enter all test results in the Excel file, you must add or remove the set numbers for the Residency and Contractor test sheets, RT and CT respectively.

Before the project starts or shortly before lot production begins, the random set numbers are determined and with at least one chosen by the Residency for each lot. These are shown below as marked out and set numbers set or not accordingly for these split samples though Residency sample can be independent after the initialization set. The Residency will compute random samples from the sublots tested by the Contractor for on-going independent validations.

Sample AC data:

Lot / Sub-Lot	Set $X_d$	Set $X_c$	AC $X_d$	AC $X_c$
1/1	1		4.1	<del>4.2</del>
1/2		2	<del>4.2</del>	4.4
1/3		3	<del>4.1</del>	4.3
1/4	4		4.2	<del>4.2</del>
2/1		5		4.2
2/2	6		4.1	<del>4.1</del>
2/3		7		4.3
2/4		8		4.2

Equation 5 – Compute the mean of the independent Contractor's results:

1. Sum the Contractor's test sets selected and divide by the total number of selected Contractor tests to find the mean.

$$\bar{X}_c = \frac{21.4}{5} = 4.280$$

Equation 6 – Compute the mean of the independent Department's results:

1. Sum the Department's test sets selected and divide by the total number of selected Department tests to find the mean.

$$\bar{X}_d = \frac{12.4}{3} = 4.133$$

Equation 7 – Compute the variance of the independent Contractor's results:

1. Compute the degrees of freedom for the Contractor tests.

$$D_f = n_c - 1 = 5 - 1 = 4$$

2. For each Contractor test result, subtract it from the average Contractor result and square the result. After summing those results, divide it by the degrees of freedom for Contractor results.

$$s_c^2 = \frac{0.028}{4} \cong 0.007$$

- a. Sample variance can also be computed in Excel by: =VAR.S(4.4,4.3,4.2,4.3,4.2)

*Equation 8 – Compute the variance of the independent Department’s results:*

1. Compute the degrees of freedom for the Department tests.

$$D_f = n_d - 1 = 3 - 1 = 2$$

2. For each Department test result, subtract it from the average Department result and square the result. After summing those results, divide it by the degrees of freedom for Department results.

$$s_d^2 = \frac{0.007}{2} \cong 0.003$$

- a. Sample variance can also be computed in Excel by: =VAR.S(4.1,4.2,4.1)

*Equation 11 – Compute the f-statistic:*

1. Compare the variances determined by equations 7 and 8. Divide the largest variance by the smallest variance to compute the f-statistic to determine if equation 9 or equation 10 governs.

$$s_c^2 > s_d^2 = 0.007 > 0.003 = TRUE \therefore F = \frac{0.007}{0.003} \cong 2.100$$

*Equation 12 or 13 – Compute f-critical, the maximum allowable f-test result:*

1. If the Department’s variance is larger than the Contractor’s variance, use equation 12 to determine f-critical. Otherwise, use equation 13.
2. In this case, as shown previously, the Contractor’s variance is larger so equation 13 governs.
  - a. The degrees of freedom for the Contractor’s number of samples as previously computed was found to be 4.
  - b. The degrees of freedom for the Department’s number of samples as previously computed was found to be 2.
  - c. The probability associated with the two-tailed F cumulative distribution, is 0.01. Probability is divided by 2 due to Excel’s one tail formula.

$$F_{crit} = F.INV.R\left(\frac{0.01}{2}, 4, 2\right) = 199.250$$

- d. For estimation purposes for this number of samples, use table 1 in Appendix D to lookup f-critical.
  - i. Since Contractor results show the highest variance in this case, the degrees of freedom for the number of Contractor independent samples govern, 4. Pick column heading “4” as the numerator.
  - ii. Consequently, the Department’s degrees of freedom in this first ongoing comparison was found to be 2. Pick “2” as the row heading number.
  - iii. The intersection of the column and row selected is the f-critical,

Critical Values of the f-statistic two tailed $\alpha = 0.01$ ,												
$D_f$ ( $n_d - 1$ or $n_c - 1$ ) for Numerator												
$D_f$ for Denominator	$n - 1$	2	3	4	5	6	7	8	9	10	11	12
	2	199.000	199.166	199.250	199.300	199.333	199.357	199.375	199.388	199.400	199.409	199.416
	3	49.799	47.467	46.195	45.392	44.838	44.434	44.126	43.882	43.686	43.524	43.387
	4	26.284	24.259	23.155	22.456	21.975	21.622	21.352	21.139	20.967	20.824	20.705
	5	18.314	16.530	15.556	14.940	14.513	14.200	13.961	13.772	13.618	13.491	13.384
	6	14.544	12.917	12.028	11.464	11.073	10.786	10.566	10.391	10.250	10.133	10.034
	7	12.404	10.882	10.050	9.522	9.155	8.885	8.678	8.514	8.380	8.270	8.176
	8	11.042	9.596	8.805	8.302	7.952	7.694	7.496	7.339	7.211	7.104	7.015

F-critical for this example's asphalt cement data by table is 199.250, as expected.

Equation 14 – Compute the pooled variance:

1. Substitute values from previous computations into the equation.

$$s_p^2 = \frac{0.007(5 - 1) + 0.003(3 - 1)}{5 + 3 - 2} \cong 0.006$$

Equation 15 – Compute the t-statistic:

1. Substitute values from previous computations into the equation.

$$t = \frac{|4.280 - 4.133|}{\sqrt{\frac{0.007}{5} + \frac{0.003}{3}}} \cong 2.642$$

Equation 16 – Compute t-critical:

1. Degrees of freedom for this first set of independent samples is 6.

$$D_f = n_d + n_c - 2 = 5 + 3 - 2 = 6$$

2. Compute t-critical:

- a. Compute t-critical using the Excel formula for a two-tailed Student's t-distribution with a probability of 0.01.

$$t_{crit} = T.INV.2T(\alpha, D_f) = T.INV.2T(0.01, 6) \cong 3.707$$

- b. For estimation purposes, use table 1 in Appendix C to lookup t-critical for this case where degrees of freedom is 6.

Critical Values of the t-statistic two tailed $\alpha = 0.01$							
$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$
2	9.925	14	2.977	26	2.779	38	2.712
3	5.841	15	2.947	27	2.771	39	2.708
4	4.604	16	2.921	28	2.763	40	2.704
5	4.032	17	2.898	29	2.756	41	2.701
6	3.707	18	2.878	30	2.750	42	2.698

*Accept Contractor's Test Results Based on OnGoing Results:*

1. If the paired t-statistic is less than or equal to t-critical, accept.

$$t \leq t_{crit} \quad 2.642 \leq 3.707 = \text{TRUE}$$

Should this comparison fail, Contractor results cannot be accepted for the lot(s) compared for validation. The cause for non-comparisons should be investigated. Until that cause is determined and this process is repeated, the Department's results will be used for acceptance.

Pay factor determinations are based on acceptance test results. Should the Contractor's results be accepted, all four of the typically PWL test sets for one lot will be utilized in pay factor determinations. In other words, consider this as a go-no-go directive for acceptance only.

## Appendix B

$$\bar{X}_p = \frac{\sum_{i=1}^{n_p} (X_c - X_d)}{n_p} \quad (\text{eq. 1})$$

Where:

- $\bar{X}_p$  = The mean (average) of the difference between paired tests;
- $n_p$  = Number of paired tests;
- $X_c$  = The Contractor's individual test result for each sample; and
- $X_d$  = Department's individual test result for each sample.

$$S_p = \sqrt{\frac{\sum_{i=1}^{n_p} (X_c - X_d - \bar{X}_p)^2}{n_p - 1}} \quad (\text{eq. 2})$$

Where:

- $S_p$  = The standard deviation of the difference between paired tests; and
- $n_p - 1$  = Number of paired tests minus one, the Degrees of Freedom ( $D_f$ ).

$$t_p = \left| \sqrt{n_p} \frac{\bar{X}_p}{S_p} \right| \quad (\text{eq.3})$$

Where:

- $t_p$  = The paired t-test statistic. Note that  $| \quad |$  is absolute the value operator.  
e.g.  $|-1.321| = 1.321$ .

$$t_{crit} = \text{T.INV.2T}(\alpha, D_f) \quad (\text{eq. 4})$$

Where:

- $t_{crit}$  = Critical or maximum allowable t-test result. Two-tailed inverse of the Student's t-distribution by Excel formula. See Appendix C table 1 for an estimate by table;
- $\alpha$  = The probability associated with the two-tailed Student's t-distribution, with a number between 0 and 1. A value of 0.01 is used; and
- $D_f$  = Degrees of Freedom for paired tests, ( $n_p - 1$ ).



$$\bar{X}_c = \frac{\sum_{i=1}^{n_c} X_c}{n_c} \quad (\text{eq. 5})$$

Where:

$\bar{X}_c$  = The mean of the independent Contractor's results;

$X_c$  = Individual Contractor result; and

$n_c$  = The number of independent Contractor results.

$$\bar{X}_d = \frac{\sum_{i=1}^{n_d} X_d}{n_d} \quad (\text{eq. 6})$$

Where:

$\bar{X}_d$  = The mean of the independent Department's results;

$X_d$  = Individual Department result; and

$n_d$  = The number of independent Department results.

$$s_c^2 = \frac{\sum_{i=1}^{n_c} (\bar{X}_c - X_c)^2}{n_c - 1} \quad (\text{eq. 7})$$

Where:

$s_c^2$  = The variance of the independent Contractor's results; and

$n_c - 1$  = Degrees of Freedom of independent Contractor results.

$$s_d^2 = \frac{\sum_{i=1}^{n_d} (\bar{X}_d - X_d)^2}{n_d - 1} \quad (\text{eq. 8})$$

Where:

$s_d^2$  = The variance of the independent Department's results (square of the sample standard deviation); and

$n_d - 1$  = Degrees of Freedom of independent Department results.

$$F_{\frac{c}{d}} = \frac{s_c^2}{s_d^2} \quad (\text{eq. 9})$$

Where:

$F_{\frac{c}{d}}$  = F-statistic when Contractor variance is greater than Department variance.

$$F_{\frac{d}{c}} = \frac{s_d^2}{s_c^2} \quad (\text{eq. 10})$$

Where:

$F_{\frac{d}{c}}$  = F-statistic when Department variance is greater than Contractor variance.

$$F = F_{\frac{c}{d}} \text{ or } F_{\frac{d}{c}} \quad (\text{eq. 11})$$

Where:

$F$  = Value of the F-statistic based on the ratio of variances where the variance value in the numerator is larger than the variance in the denominator.

$$F_{crit} = \text{F.INV.R} \left( \frac{\alpha}{2}, n_d - 1, n_c - 1 \right) \quad (\text{eq. 12})$$

Where:

$F_{crit}$  = Critical or maximum allowable f-test result. Two-tailed F cumulative distribution by Excel formula. See Appendix D for an estimate by table. Note that  $n_d - 1$  is used first if the Department variance is higher than Contractor variance. Otherwise, replace  $n_d$  with  $n_c$  and vice-versa in this equation as shown in equation 13.

$$F_{crit} = \text{F.INV.R} \left( \frac{\alpha}{2}, n_c - 1, n_d - 1 \right) \quad (\text{eq. 13})$$

$$s_p^2 = \frac{s_c^2(n_c - 1) + s_d^2(n_d - 1)}{n_c + n_d - 2} \quad (\text{eq. 14})$$

Where:

$s_p^2$  = The pooled variance.

$$t = \frac{|\bar{X}_c - \bar{X}_d|}{\sqrt{\frac{s_p^2}{n_c} + \frac{s_p^2}{n_d}}} \quad (\text{eq. 15})$$

Where:

$t$  = The t-statistic.

$$t_{crit} = T.INV.2T(\alpha, D_f) \quad (\text{eq. 16})$$

Where:

- $t_{crit}$  = Critical or maximum allowable t-test result. Two-tailed inverse of the Student's t-distribution by Excel formula. See Appendix C for an estimate by table; and
- $D_f$  = Degrees of Freedom for independent Department and Contractor tests,  $(n_d + n_c - 2)$ .

## Appendix C

Critical Values of the t-statistic two tailed $\alpha = 0.01$							
$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$	$D_f$ $n_p - 1$ or $n_d + n_c - 2$	$t_{crit}$
2	9.925	14	2.977	26	2.779	38	2.712
3	5.841	15	2.947	27	2.771	39	2.708
4	4.604	16	2.921	28	2.763	40	2.704
5	4.032	17	2.898	29	2.756	41	2.701
6	3.707	18	2.878	30	2.750	42	2.698
7	3.499	19	2.861	31	2.744	43	2.695
8	3.355	20	2.845	32	2.738	44	2.692
9	3.250	21	2.831	33	2.733	45	2.690
10	3.169	22	2.819	34	2.728	46	2.687
11	3.106	23	2.807	35	2.724	47	2.685
12	3.055	24	2.797	36	2.719	48	2.682
13	3.012	25	2.787	37	2.715	49	2.680

**Table 1**

# Appendix D

Critical Values of the f-statistic two tailed $\alpha = 0.01$ ,													
$D_f (n_d - 1 \text{ or } n_c - 1)$ for Numerator													
$D_f$ for Denominator	$n - 1$	2	3	4	5	6	7	8	9	10	11	12	
	2	199.000	199.166	199.250	199.300	199.333	199.357	199.375	199.388	199.400	199.409	199.416	
	3	49.799	47.467	46.195	45.392	44.838	44.434	44.126	43.882	43.686	43.524	43.387	
	4	26.284	24.259	23.155	22.456	21.975	21.622	21.352	21.139	20.967	20.824	20.705	
	5	18.314	16.530	15.556	14.940	14.513	14.200	13.961	13.772	13.618	13.491	13.384	
	6	14.544	12.917	12.028	11.464	11.073	10.786	10.566	10.391	10.250	10.133	10.034	
	7	12.404	10.882	10.050	9.522	9.155	8.885	8.678	8.514	8.380	8.270	8.176	
	8	11.042	9.596	8.805	8.302	7.952	7.694	7.496	7.339	7.211	7.104	7.015	
	9	10.107	8.717	7.956	7.471	7.134	6.885	6.693	6.541	6.417	6.314	6.227	
	10	9.427	8.081	7.343	6.872	6.545	6.302	6.116	5.968	5.847	5.746	5.661	
	11	8.912	7.600	6.881	6.422	6.102	5.865	5.682	5.537	5.418	5.320	5.236	
	12	8.510	7.226	6.521	6.071	5.757	5.525	5.345	5.202	5.085	4.988	4.906	

**Table 1**

Critical Values of the f-statistic two tailed $\alpha = 0.01$ ,													
$D_f (n_d - 1 \text{ or } n_c - 1)$ for Numerator													
$D_f$ for Denominator	$n - 1$	13	14	15	16	17	18	19	20	21	22	23	
	2	199.423	199.428	199.433	199.437	199.441	199.444	199.447	199.450	199.452	199.454	199.456	
	3	43.271	43.172	43.085	43.008	42.941	42.880	42.826	42.778	42.733	42.693	42.656	
	4	20.603	20.515	20.438	20.371	20.311	20.258	20.210	20.167	20.128	20.093	20.060	
	5	13.293	13.215	13.146	13.086	13.033	12.985	12.942	12.903	12.868	12.836	12.807	
	6	9.950	9.877	9.814	9.758	9.709	9.664	9.625	9.589	9.556	9.526	9.499	
	7	8.097	8.028	7.968	7.915	7.868	7.826	7.788	7.754	7.723	7.695	7.669	
	8	6.938	6.872	6.814	6.763	6.718	6.678	6.641	6.608	6.578	6.551	6.526	
	9	6.153	6.089	6.032	5.983	5.939	5.899	5.864	5.832	5.803	5.776	5.752	
	10	5.589	5.526	5.471	5.422	5.379	5.340	5.305	5.274	5.245	5.219	5.195	
	11	5.165	5.103	5.049	5.001	4.959	4.921	4.886	4.855	4.827	4.801	4.778	
	12	4.836	4.775	4.721	4.674	4.632	4.595	4.561	4.530	4.502	4.476	4.453	

**Table 2**

Critical Values of the f-statistic two tailed $\alpha = 0.01$ ,													
$D_f (n_d - 1 \text{ or } n_c - 1)$ for Numerator													
$D_f$ for Denominator	$n - 1$	24	25	26	27	28	29	30	31	32	33	34	
	2	199.458	199.460	199.461	199.463	199.464	199.465	199.466	199.467	199.468	199.469	199.470	
	3	42.622	42.591	42.562	42.535	42.511	42.487	42.466	42.446	42.427	42.409	42.392	
	4	20.030	20.002	19.977	19.953	19.931	19.911	19.892	19.874	19.857	19.841	19.826	
	5	12.780	12.755	12.732	12.711	12.691	12.673	12.656	12.639	12.624	12.610	12.597	
	6	9.474	9.451	9.430	9.410	9.392	9.374	9.358	9.343	9.329	9.316	9.303	
	7	7.645	7.623	7.603	7.584	7.566	7.550	7.534	7.520	7.507	7.494	7.482	
	8	6.503	6.482	6.462	6.444	6.427	6.411	6.396	6.382	6.369	6.357	6.345	
	9	5.729	5.708	5.689	5.671	5.655	5.639	5.625	5.611	5.598	5.586	5.575	
	10	5.173	5.153	5.134	5.116	5.100	5.085	5.071	5.057	5.045	5.033	5.022	
	11	4.756	4.736	4.717	4.700	4.684	4.668	4.654	4.641	4.629	4.617	4.606	
	12	4.431	4.412	4.393	4.376	4.360	4.345	4.331	4.318	4.305	4.294	4.283	

**Table 3**

Critical Values of the f-statistic two tailed $\alpha = 0.01$ ,												
$D_f (n_d - 1 \text{ or } n_c - 1)$ for Numerator												
$D_f$ for Denominator	$n - 1$	2	3	4	5	6	7	8	9	10	11	12
	13	8.186	6.926	6.233	5.791	5.482	5.253	5.076	4.935	4.820	4.724	4.643
	14	7.922	6.680	5.998	5.562	5.257	5.031	4.857	4.717	4.603	4.508	4.428
	15	7.701	6.476	5.803	5.372	5.071	4.847	4.674	4.536	4.424	4.329	4.250
	16	7.514	6.303	5.638	5.212	4.913	4.692	4.521	4.384	4.272	4.179	4.099
	17	7.354	6.156	5.497	5.075	4.779	4.559	4.389	4.254	4.142	4.050	3.971
	18	7.215	6.028	5.375	4.956	4.663	4.445	4.276	4.141	4.030	3.938	3.860
	19	7.093	5.916	5.268	4.853	4.561	4.345	4.177	4.043	3.933	3.841	3.763
	20	6.986	5.818	5.174	4.762	4.472	4.257	4.090	3.956	3.847	3.756	3.678
	21	6.891	5.730	5.091	4.681	4.393	4.179	4.013	3.880	3.771	3.680	3.602
	22	6.806	5.652	5.017	4.609	4.322	4.109	3.944	3.812	3.703	3.612	3.535
23	6.730	5.582	4.950	4.544	4.259	4.047	3.882	3.750	3.642	3.551	3.475	

**Table 4**

Critical Values of the f-statistic two tailed $\alpha = 0.01$ ,												
$D_f (n_d - 1 \text{ or } n_c - 1)$ for Numerator												
$D_f$ for Denominator	$n - 1$	2	3	4	5	6	7	8	9	10	11	12
	24	6.661	5.519	4.890	4.486	4.202	3.991	3.826	3.695	3.587	3.497	3.420
	25	6.598	5.462	4.835	4.433	4.150	3.939	3.776	3.645	3.537	3.447	3.370
	26	6.541	5.409	4.785	4.384	4.103	3.893	3.730	3.599	3.492	3.402	3.325
	27	6.489	5.361	4.740	4.340	4.059	3.850	3.687	3.557	3.450	3.360	3.284
	28	6.440	5.317	4.698	4.300	4.020	3.811	3.649	3.519	3.412	3.322	3.246
	29	6.396	5.276	4.659	4.262	3.983	3.775	3.613	3.483	3.377	3.287	3.211
	30	6.355	5.239	4.623	4.228	3.949	3.742	3.580	3.450	3.344	3.255	3.179
	31	6.317	5.204	4.590	4.196	3.918	3.711	3.549	3.420	3.314	3.225	3.149
	32	6.281	5.171	4.559	4.166	3.889	3.682	3.521	3.392	3.286	3.197	3.121
	33	6.248	5.141	4.531	4.138	3.861	3.655	3.495	3.366	3.260	3.171	3.095
34	6.217	5.113	4.504	4.112	3.836	3.630	3.470	3.341	3.235	3.146	3.071	

**Table 5**